Thoughts on Causal Inference in Games

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What is this lecture about?

- I will try to make connections between the "causal inference" literature in statistics and economics to objects of inference in the empirical games literature and the role **simultaneity** plays.

- In statistics (and some economics), Causal inference is tied to an action applied to a unit (intervention, treatment, manipulation) - it can be a mental exercise. The question is narrowly defined and is one of evaluation (is one treatment better than another).

- In economics and in games, we also posit **response functions** -production functions, demand, best response etc- that can answer similar questions. The "causal effect" is defined implicitly through these functions.

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- However, “learning about the science” here can differ drastically because of simultaneity and its relation to data. This is the objective of the talk.

Plan

- I will review the "counterfactual notation" in causal inference, and what the science is vs what we can do to learn about the science.
- This is well within what economists had been doing and continue to do.
- Then, I will illustrate that the exercise gets complicated in games setups when simultaneity is present (multiple decision makers).
- This links well also to the recent works on "social interactions models" (i.e. games), and other "network like" models.

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Objects of Interest

We care about **Response Functions:**

- Those are known in the causal literature as **Potential Functions** which are functions mapping treatments to outcomes.

- In games, we use best response functions: demand, supply, or production functions, etc.

- A crucial element is **assignment mechanism** or **equilibrium model** that tells the story.

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The role of time:

- A foundational piece in the "causal model" is the timing implicit between treatment and outcomes:

"Let $Y(E)$ be the value of Y measured at time t_2 on the unit given that the unit received the control Treatment E initiated at time t_1 where $t_1 < t_2$"

— Rubin (1974)

- This is not always the case in games: Demand functions, best response functions, etc where "treatments" are simultaneous with outcomes.

in games... with simultaneity

- Problems arise when observations are of equilibrium outcomes as in models with social interactions

- For example: if the object of interest is the distribution of demand and supply functions $P[s(.), d(.)|x]$: Suppose that we observe, by some sampling process of markets say, the distribution $P(p, q, x)$. This is not sufficient for identification of $P[s(.), d(.)|x]$. In fact, it is not even clear that at data point (p_i, q_i, x_i), we have $q_i = s(p_i)$ without further assumptions.

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Examples

Consider the following model: Suppose we have a set of iid observations on outcomes, treatments from N units $(Y_i, T_i), i = 1, \ldots, N$.

The only requirement here is that T_i precedes Y_i

Examples:

- vaccination campaign $\mapsto$ Health outcomes
- class assignment $\mapsto$ test scores
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Assuming random assignment of T (binary):

Here, we have

- at the observed pair (Y, T):

$$Y \equiv Y(T = t) = f(T = t, \epsilon_i)$$

- $E[Y(1)] - E(Y(0)) = E(Y|T = 1) - E(Y|T = 0) = Ef(1, \epsilon) - Ef(0, \epsilon)$
- The framework is agnostic about what happens betweeen: multiple equilibria, etc. For example, suppose that $T_i = 1$ means that household i receives a vaccination subsidy in village i at random. Suppose that after the receipt of treatment, some households play a game of whether to go out and buy vaccine and the game has multiple equilibria (this can be made formal). As long as we have exchangeability, a sample allows us to obtain the average as N is large via Di Finetti's Theorem.

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- Here, we can think of $\mathbf{T}$ as an assignment *vector*: which household in village i gets a subsidy to buy vaccine, and which one does not.

- It is likely that once T is assigned, households play a "game" that may have multiple equilibria: for example in a coordination game, two pure strategy NE arise: everyone gets vaccinated, or no one does. There is also a mixed strategy equilibrium.

- However, learning the ATE from an exchangeable sequence of villages is still possible regardless of what happens in between. *Social interaction, games etc complicates the explanation, but not the mechanics of learning.*

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Without Random Assignment

- We can always use Worst Case Bounds etc.
- Or, we can posit a choice model along with an instrument Z:

  1. Choice Model: $T_i = g(Z_i, V_i)$ where V_i is unobservable and $g(.)$ the choice mapping,
  2. (i) Exclusion: $Y(t, z) = Y(t)$ and (ii) random assignment of Z:

  $$\{(Y(t) : t = 1, ...), V\} \perp Z$$

- Claim: Wald Estimator does not converge to a meaningful coefficient.

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the single agent scalar V case

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reduced form ↦ causal parameters

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\begin{aligned}
E[Y|Z=z] &= E\left[E[Y(1)-Y(0)|V]1[V \leq g(z)]\right] \\
&= \int_0^{g(z)} E[Y(1)-Y(0)|V]dV \\
P(T=1|Z=z) &= g(z)
\end{aligned}
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It is clear then that $(g(z_0) \leq g(z_1))$:

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So,

$$\frac{E[Y|z_1] - E[Y|z_0]}{g(z_1) - g(z_0)} = E[Y(1)-Y(0)|g(z_0) \leq V \leq g(z_1)|]$$

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The key here is that the monotonicity (V *is a scalar*) divides *types* (where V lies with respect to $[g(z_0), g(z_1)]$) in a particularly *overlapping way*: types that choose 1 when $Z = z_1$ "cover" the region of types that choose 1 when $Z = z_0$ (this overlapping regions idea can be -and should be- formalized).

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Now when the choice model is a game:

Again, now: suppose that we are interested in the causal effect of vaccination on health outcomes in a sample of villages (ex: $E[Y(1) - Y(0)]$). The treatment now is a vector $\mathbf{T}$.

The choice problem now is a game:

$$T = G(Z, T, \epsilon)$$

Theorem

The Wald estimator can be negative even though the treatment effect for every individual is positive.

Multiple equilibria violates "monotonicity" even if we have a univariate unobservable. The above Theorem is based on multiplicity generating problematic overlapping regions where regions for various types have nonempty set difference.

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Conclusion so far:

Good news: If treatment comes at time t_0 and outcomes are realized at time t_1 where $t_0 < t_1$, we can still get ATE in a randomized experiment with games played -and multiplicity- when we have access to an exchangeable data set.

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Causal Models when Treatment and Outcomes are Simultaneous

Examples:

- Demand and Supply functions $(p_d(t), p_s(t))$: treatment is quantity and outcome is price (or vice versa). Again, we get to observe $P(p, q)$ or the distribution of market realizations of (p, q).

- Models of Games where the observation of **equilibrium** outcomes are used to identify players' **reaction functions:** Here, "treatments" are other player's actions and outcomes are your actions.

- Models of networks, social effects, etc.

This setting is problematic for the standard causal inference literature. Now, even ATE's are not automatic. I will illustrate next with a simple binary game. This is similar to work I have done with Kline.

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Binary Games

- Suppose we have a **random sample of outcomes** from binary (entry-like) games. These observed outcomes are pairs (a_1, a_2) where $a_i \in \{0, 1\}$ for $i = 1, 2$.

- We are interested in learning about $P[y_1(1) = 1]$ and $P[y_2(2) = 1]$ using our knowledge of $P(a_1, a_2)$ where $y_i(.)$ is the best response function of player i.

- Think of $y_i(a)$ as the response function for player i evaluated at action a of opponent (what is my best response to my opponent entering the market)

- The **key problem** is the link between observed outcomes and underlying best responses.

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- The **key problem** is the link between observed outcomes and underlying best responses.

- What does observing (a_1, a_2) tell us about response functions in the presence of multiplicity, mixed strategies, non-equilibrium play, assumptions whether players know others' utility function, etc.

- **Key Insight:** It is NOT true now that observing (a_1, a_2) implies that we observe the best response functions: $y_1(a_2) = a_1$. This was taken as given in the case when treatments precede outcomes ($Y_i = Y_i(1)T_i + (1 - T_i)Y_i(0)$)

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Consider the following game:

	0	1
0	0,0	0,π_0^2
1	π_0^1,0	π_1^1, π_1^2

- Random Sampling: we have an iid sample $(a_1^i, a_2^i), \quad i = 1, \ldots, N$ where $a_i^j \in \{0, 1\}$ for j=1,2.

- Econometricians do not observe the $\pi's$.

- Interested in the distribution $P\left[y_1(1) = 1\right]$ or the distribution of player 1's best response to player 2 playing 1. This is the treatment response.

Identification of $P(y_1(1) = 1)$

Using the law of total probability, the best response functions:

$$\begin{aligned} P(y_1(1) = 1) &= P(\pi_1^1 \geq 0) \\ &= \underbrace{P(\pi_1^1 \geq 0 | (1,1))}_{(1)} P(1,1) + \underbrace{P(\pi_1^1 \geq 0 | (0,0))}_{(2)} P(0,0) \\ &\quad + \underbrace{P(\pi_1^1 \geq 0 | (1,0)}_{(3)} P(1,0) + \underbrace{P(\pi_1^1 \geq 0 | (0,1))}_{(4)} P(0,1) \end{aligned}$$

The unconditional probabilities above are all identified from the data.

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Start with (1) above

$(1) : P(\pi_1^1 \geq 0 | (1,1))$

- Observing $(1,1)$, means that playing 1 is a best response for player 1 to some strategy of player 2.

- This means that for *some p* of the opponents possible **mixed strategies** we must have

$$p\pi_0^1 + (1-p)\pi_1^1 \geq 0$$

So, you can have π_1^1 be negative or positive, and hence

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Identification of $P(y_1(1) = 1)$: (2) above

Again, here we need to bound $P(\pi_1^1 \geq 0|(0,0))$. by arguments similar to the above, if we allow for mixed strategies, then we can show that

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The same applies to both (3) and (4).

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Theorem

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We need to add assumptions beyond the standard:

Theorem

1. *With monotonicity,* $\pi_0^i \geq \pi_1^i \quad i = 1, 2$*, then*

$$P(y_1(1) = 1) \in [0, P(a_1 = 1)]$$

   These bounds are sharp even if we assume L_1 *rationality only.*

2. *Under Symmetry of payoffs, monotonicity and* L_1*, we have*

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Assume monotonicity. The sharp joint identification region for $P(y_1(1)) = 1$ and $P(y_2(1)) = 1$ is:

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Take Aways

- Even in *binary games* with complete information (simplest setup I can think of) causal analysis using the "counterfactual notation" is useless without thinking of an economic model,
- Shape restrictions matter (symmetry, monotonicity), *but also* behavior. Are agents rational? how so? (L_1, L_2, rationalizable strategies, etc...)
- One needs to think about the various interpretations of the response functions within the framework of a model.
- But, of course, the Rubin style causal model did not allow for simultaneity. And as we see here, for good reason.

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Linear in Means Model: What is going on?

The linear-in-means *model* posits that for a group of N individuals, we have

$$\mathbf{Y} = \beta_0 \iota_N + \mathbf{X}\beta_X + \beta_{\overline{y}}\mathbf{GY} + \mathbf{GX}\beta_{\overline{x}} + \eta,$$

- $\mathbf{Y}$ is the $N \times 1$ vector of outcomes,
- ι_N is the $N \times 1$ vector of ones,
- $\mathbf{X}$ is the $N \times K$ matrix of observable covariates,
- $\mathbf{G}$ is the $N \times N$ row-normalized adjacency matrix, and
- η is the $N \times 1$ vector of unobservables.

The parameters β_0, β_x, $\beta_{\overline{y}}$, and $\beta_{\overline{x}}$ are the unknown parameters of the model, which are 1×1, $K \times 1$, 1×1, and $K \times 1$, respectively.

What do we get from this model, and how do we interpret its coefficients?

How do we relate *some causal effect* to the above parameters? what is the relationship of this to games?

First View:

- One can view the above as a particular example of a simultaneous equation system, and *a causal effect* of interest may be the impact of variables x.

  Think of x here as randomly assigned (so no identification issues - though this can be relaxed).

- Here, if units are treated with x *prior* to outcomes, then as we saw above, we can estimate the average treatment effect directly.

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- The parameters above can also be derived exactly as parameters of some utility function and the model becomes one of a game with the linear in means model being a best response function.

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Causal effect of a change in x in the above

LIM Causal effect of $\mathbf{X}$ (under random assignment) on $\mathbf{Y}$, that accounts for the re-calibration of the outcomes $\mathbf{Y}$ after an exogenous change in $\mathbf{X}$, and its relation to "beta."

Theorem (LIM Causal Effect)

Suppose model above holds with $E(\eta|\mathbf{X},\mathbf{G}) = 0$. Suppose that $\Upsilon \equiv \mathbf{I}_{N\times N} - \beta_{\overline{y}}\mathbf{G}$ is non-singular. Let e_i be the $1 \times N$ unit row vector with a 1 in the ith column. We have

1. *In general, the marginal effects of $\mathbf{X}$ on $E(y_i|\mathbf{X},\mathbf{G})$ are given by $\beta_x e_i \Upsilon^{-1} + \beta_{\overline{x}} e_i \Upsilon^{-1}\mathbf{G}$, where the marginal effect of the k-th covariate of individual j is the (k,j) entry of this matrix.*
2. *In the special case that $\mathbf{G}$ represents the complete network, the effects of the various elements of $\mathbf{X}$ on $E(y_i|\mathbf{X},\mathbf{G})$ are the following:*
   - 2.1 *The effect of x_{ik} is $\lambda\beta_{xk} + \psi\beta_{\overline{x}k}$*
   - 2.2 *The effect of x_{jk} is $\psi\beta_{xk} + \omega\beta_{\overline{x}k}$ for $j \neq i$ where $\lambda = \frac{1-N+\beta_{\overline{y}}(N-2)}{(\beta_{\overline{y}}-1)(N-1+\beta_{\overline{y}})}$, $\psi = \frac{-\beta_{\overline{y}}}{(\beta_{\overline{y}}-1)(N-1+\beta_{\overline{y}})}$, and $\omega = \frac{-1}{(\beta_{\overline{y}}-1)(N-1+\beta_{\overline{y}})}$.*

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Remarks

1. A sufficient condition for non-singularity of Υ is $|\beta_{\overline{y}}| < 1$. Note that Υ is the matrix with 1s along the diagonal, and that the row sum of the absolute values of the off-diagonal entries is $|\beta_{\overline{y}}|$. If $|\beta_{\overline{y}}| < 1$, Υ is strictly diagonally dominant, so has an inverse.
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3. The effect of changing the covariates of individual i's peers on the outcome of individual i is generically *not* given by the parameter $\beta_{\overline{x}}$, because the change in covariates also has an effect on outcomes, which is intermediated by the social interaction in the outcomes.

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Takeaway here...

If you insist on the model above, and are interested in the causal impact of a variable x, then this effect is a complicated function of the parameters.

It is simple for example to show that β_x necessarily understates the magnitude of these effects except in the special case of $\beta_y = 0$, and indeed may understate the effects by an arbitrarily large ratio, depending on the value of β_y .

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Interpretation of above as a game

- There is a way to interpret the above as the best response function from some game.
- The above regression can be seen as a *response function* where the outcome of others *are held constant and are nonrandom* and so it is not necessary to condition on them.
- This allows extrapolating from observations of (some) equilibrium behavior to draw conclusions about responses to manipulating an individual's peer outcomes (off-equilibrium).
- The extra *assumption* of the linear in means game (one such game is defined next) is necessary for this interpretation (just as in the demand and supply *model* which presupposes the existence of demand/supply functions that are well defined even at non-equilibrium quantities. This is called "autonomy" and is discussed by Goldberger for example).

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But, the interpretation as best response requires a model:

Theorem

Suppose that individual i with type θ_i has utility function $u_i(y_i, y_{-i}; \theta_i)$.
Suppose that $u_i(\cdot)$ is a polynomial in y_i, and in particular is such that maximizing $u_i(\cdot)$ with respect to y_i follows from the usual first and second order conditions, and $\frac{\partial^2 u_i}{\partial y_i^2} < 0$.
And suppose that FOC has the unique solution $y_i = \theta_i + \frac{\phi}{N_g - 1}\sum_{j \neq i} g_j y_j = x_i\beta + x_g\beta_x + \frac{\phi}{N_g - 1}\sum_{j \neq i} g_j y_j + \eta_i$ where $\theta_i = x_i\beta + x_g\beta_x + \eta_i$
Then it must be that $u_i(y_i, y_{-i})$ is equal to $\theta_i y_i - \frac{y_i^2}{2} + \frac{\phi}{N_g - 1}\sum_{j \neq i} y_j$ up to positive affine transformation, where the constants in the transformation may depend on y_{-i} and θ but not y_i, and so in particular for any y_{-i} individual i has an incentive to conform to $\frac{1}{N_g - 1}\sum_{j \neq i} g_j y_j$.

But, the interpretation as best response requires a model:

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Suppose that individual i with type θ_i has utility function $u_i(y_i, y_{-i}; \theta_i)$. Suppose that $u_i(\cdot)$ is a polynomial in y_i, and in particular is such that maximizing $u_i(\cdot)$ with respect to y_i follows from the usual first and second order conditions, and $\frac{\partial^2 u_i}{\partial y_i^2} < 0$.

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Comments on Game Interpretation

- The positives about assuming the above game is that when we learn about the parameters, we are learning about the utilities directly. Also, it makes sense to talk about what happens when we randomize or manipulate one's peers. Here, a "causal effect" in the game above can be clearly defined.

- Also, the assumptions that data are realizations from the intersection of best responses implicitly entails the assumptions of pure strategy NE. But, NE is not always a reasonable characterization of behavior...

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- If treatments are not simultaneous with outcomes, then there are no problems beyond identification (the science is fine, learning about the science may need some care with multiplicity causing non-overlapping sets of types - abandon the Wald statistic)
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