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RESERVE PRICES IN AUCTIONS AS REFERENCE POINTS*

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We consider second-price and first-price auctions in the symmetric independent private values framework. We modify the standard model by the assumption that the bidders have reference-based utility, where a publicly announced reserve price has some influence on the reference point. It turns out that the seller's optimal reserve price increases with the number of bidders. Also in contrast to the standard model, we find that secret reserve prices can outperform public reserve prices, and that setting the optimal reserve price can be more valuable for the seller than attracting additional bidders.

In recent years, theorists have begun to use the standard tools of microeconomics to explore the implications of assumptions on human behaviour based on insights imported from psychology.[1] One of the most prominent departures from the standard economic paradigm is the assumption that people have reference-based utility; i.e., they assess utilities in comparison with reference points (Kahneman and Tversky, 1979; Tversky and Kahneman, 1991). In this article, we investigate how the analysis of the standard model of second-price and first-price auctions changes if the reserve price (i.e., minimum bid) announced by the seller has some (possibly very small) influence on what the potential buyers perceive as a reference point. It turns out that our model has interesting implications that may help to explain features of real auctions (e.g., secret reserve prices) that have escaped the standard analysis.

We consider the well-known symmetric independent private values model with risk-neutral agents, which is the simplest framework in which auctions have been analysed. We assume that if a bidder wins the object and has to make a payment t, then his utility is given by $v - t - \varepsilon(t - \rho)$, where v is the bidder's intrinsic valuation, ρ is the reference point, and ε is a small positive number. The case $\varepsilon = 0$ corresponds to the standard model in the auction literature. We are interested in the implications of the case $\varepsilon > 0$, which captures the disutility (or utility) that a buyer perceives if he has to pay more (or less) than the reference point. The reference point can depend on various exogenous parameters such as selling prices in auctions of related items, estimates delivered by auction house experts etc. Yet, if the seller publicly announces a reserve price, it is plausible to assume that this announcement also has some influence on the reference point. Empirically, the fact that reserve prices in auctions are indeed perceived by bidders as reference points has recently been shown in field studies as well as in laboratory experiments (Häubl and Popkowski Leszczyc, 2003; Ariely and Simonson, 2003; Kamins *et al.*, 2004).[2]

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[1] In a recent survey article, Rabin (2002) has called this new movement 'second-wave behavioural economics', because it goes beyond simply pointing out problems with standard economic assumptions.

[2] In their field study, Häubl and Popkowski Leszczyc (2003) emphasise that the seller-specified reserve price remains to be relevant, even if objective reference prices such as published catalogue values are available.

Hence, we model the reference point as a convex combination of an exogenous parameter x and the reserve price r, where the weight that is associated with the reserve price is positive but maybe very small.

It turns out that in our setting with publicly announced reserve prices, first-price and second-price auctions are revenue-equivalent, which is in accordance with the standard model. Yet, a remarkable result of the standard analysis is that the optimal reserve price does not depend on the number of bidders (Myerson, 1981; Riley and Samuelson, 1981). In contrast, we will show that if there is a reference-point effect, then the optimal reserve price increases with the number of bidders. In our setting, the reserve price has a positive effect on a bidder's willingness-to-pay. Of course, making the reserve price larger always has the disadvantage of increasing the probability that there will be no trade, but this probability decreases with the number of bidders. Hence, the optimal reserve price will rise if the number of bidders goes up. This finding is consistent with a recent empirical study conducted by Reiley (2006), who reports results from a field experiment suggesting that the optimal reserve price may indeed be increasing with the number of bidders.[3]

In practice, reserve prices are often kept secret. In his description of how real auctions work for wine and art, Ashenfelter (1989) points out that auction houses such as Christie's and Sotheby's usually do not reveal reserve prices. In a study of online auctions, Bajari and Hortaçsu (2003) find that secret reserve prices may lead to larger revenues than public minimum bids set at the same level.[4] These observations are a puzzle from the perspective of the standard analysis.[5] In a second-price auction, it should not matter whether or not a reserve price is kept secret – the bidders always have a dominant strategy to bid their true valuation, see Riley and Samuelson (1981) – while in a first-price auction, the seller's revenue is in general strictly larger if the reserve price is made public (Elyakime *et al.*, 1994).[6] In contrast, in our model the seller's revenue may well be larger if she keeps the reserve price secret, both in second-price and first-price auctions. Intuitively, if the exogenous factors that influence the reference point are relatively large, then the announcement of a reserve price (which has to be small enough so that the no-trade outcome is not triggered too often) reduces the reference point and hence the bidders' willingness-to-pay.[7]

Another novel conclusion that can be drawn from our analysis deals with the relative advantages of auctions and negotiations as discussed by Bulow and

[3] Reiley (2006) points out that more experiments are needed to provide cleaner tests of this hypothesis, which we also consider to be very desirable in the light of the novel theory that we are proposing.

[4] Also studying online auctions, Katkar and Reiley (2005) find that keeping reserve prices secret can make sellers worse off. The ambiguity in the empirical results is compatible with our theory, since in our model it will depend on the parameter constellation whether or not secret reserve prices are more profitable than public reserve prices.

[5] See, however, Vincent (1995) and Horstmann and LaCasse (1997), who argue that under certain circumstances secret reserve prices may be advantageous in common-value auctions.

[6] In a first-price auction, the optimal bids depend on the reserve price. Intuitively, not making the reserve price public is as if it were chosen simultaneously with the buyers' bids; i.e., the seller just does not make use of the fact that she can be a 'Stackelberg leader'.

[7] In the empirical literature, Kaiser and Kaiser (1999) and Bajari and Hortaçsu (2003) report that secret reserve prices seem to be more useful to sellers when the goods being auctioned have higher book values. This empirical regularity is consistent with our theory when higher book values are reflected in relatively larger values of x.

Klemperer (1996). They argue that an auction with no reserve price and $n+1$ bidders is always more profitable than an optimally-structured negotiation (modelled as an auction with an optimal reserve price) with n bidders. Hence, standard models cannot explain why negotiations are sometimes restricted to a few bidders even if this allows the seller to maintain control of the negotiation process (i.e., to commit credibly to a reserve price). In other words, even if dealing with more bidders means that the seller loses her commitment power, in the standard model she is always better off with at least one additional bidder. We show that this result no longer holds in the presence of a reference point effect. Our model has the intuitive property that an additional bidder can be less valuable for the seller than her ability to structure the selling mechanism optimally. Thus, in contrast to the standard theory, our model is consistent with 'lock-up' agreements that rule out negotiations with additional potential acquirers.[8]

To the best of our knowledge, this is the first article in the literature on auction theory in which the implications of reference-based utility are explored in a formal model. Our article makes a contribution to a growing literature that incorporates behavioural assumptions into standard economic analysis and studies their consequences. Papers that are thus related in spirit include the recent work on the implications of inequity aversion, fairness, ethics, and honesty in agency and mechanism design theory (Alger and Ma, 2003; Alger and Renault, 2006; Chen, 2000; Deneckere and Severinov, 2003; Matsushima, 2002).[9]

The remainder of the article is organised as follows. In Section 1, the model is introduced and the equilibrium strategies in second-price and first-price auctions are discussed. In Section 2, the seller's optimal reserve price is characterised and comparative statics results are provided. Our model is applied in order to derive novel results regarding secret reserve prices and the value of additional bidders in Section 3. Concluding remarks follow in Section 4. Some technical details have been relegated to the Appendix.

1. The Model

Consider a monopolistic seller who has a single, indivisible object for sale, that she cannot use herself. There are n potential buyers. The seller conducts a (second-price or first-price) sealed-bid auction with reserve price r, which means that a bidder participating in the auction must bid at least r. If buyer i does not win the auction, his utility is given by zero. If buyer i wins the object and must pay the price t_i according to the rules of the auction, then his utility is given by $v_i - t_i - \varepsilon(t_i - \rho)$, where $v_i \in [0, 1]$ denotes his intrinsic valuation. The case $\varepsilon = 0$ is the usual case analysed in the auction literature. A positive (but possibly very small) ε captures the reference point effect as discussed in the introduction. Specifically, the reference point is given by $\rho = \lambda r + (1 - \lambda)x$, where $x \in [0, 1]$ is an exogenous parameter (e.g., reflecting selling prices in auctions of related items, estimates delivered by

[8] As Bulow and Klemperer (1996) concede, under dominant US takeover law 'lock-up' provisions are in fact allowed if the board is acting in the shareholders' interests and the price attained is high enough.

[9] For further references on psychology and economics, see also Tirole (2002), Rabin (2002), Camerer and Loewenstein (2003) and Fehr and Schmidt (2003).

auction house experts etc.) and $\lambda \in [0, 1]$ denotes the weight attached to the announced reserve price.

Buyer i's type v_i is the realisation of a random variable $\tilde{v}_i$. Each $\tilde{v}_i$ is independently and identically distributed on the unit interval. The distribution function F is strictly increasing and the differentiable density function is denoted by f. Moreover, we make the usual monotone hazard rate assumption, so that

$$\frac{1 - F(v)}{f(v)}$$

is decreasing in v.[10] Only buyer i knows his realised value v_i, while the other components of the model are assumed to be common knowledge. Each agent is interested in maximising his or her expected payoff. Hence, our analysis is directly comparable with the standard model of the independent private values environment with symmetric bidders as analysed by Riley and Samuelson (1981).[11]

1.1. *Second-Price Auction*

In a second-price auction in which at least two bidders participate, the buyer submitting the highest bid wins the object, but he has to pay only the second-highest bid.[12] If only one bidder participates, he wins and has to pay the reserve price r. It is well known that in the standard case ($\varepsilon = 0$), each buyer i with $v_i \geq r$ will participate in the auction and bid his type v_i. In the present framework with $\varepsilon \geq 0$, this result can be generalised as follows.

PROPOSITION 1 *In a second-price auction, it is a weakly dominant strategy for a buyer of type v to bid*

$$b^S(v) = \frac{v + \varepsilon\rho}{1 + \varepsilon}$$

if $v \geq \bar{v}(r) = (1 + \varepsilon)r - \varepsilon\rho$, and not to participate otherwise.

Proof. Since the price for the object will at least be r, it cannot be profitable for buyer i to participate if $v_i - r - \varepsilon(r - \rho) < 0$. Thus, consider a buyer i with $v_i \geq \bar{v}(r)$. If buyer i bids $b^S(v_i)$, he wins if $b^S(v_i) > t_i$, where t_i is the maximum of the other bids if there are any, and $t_i = r$ otherwise. Consider a downward deviation to some $\tilde{b} < b^S(v_i)$. If $t_i < \tilde{b} < b^S(v_i)$, he still wins and pays t_i. If $\tilde{b} < b^S(v_i) \leq t_i$, his payoff is still zero. If $\tilde{b} < t_i < b^S(v_i)$, he now loses and gets zero, while he would have made a profit $v_i - t_i - \varepsilon(t_i - \rho)$ by bidding $b^S(v_i)$. This profit would have been positive, since

[10] Hence, we are in Myerson's (1981) 'regular case', i.e. the 'virtual valuation' $v - [1 - F(v)]/f(v)$ is increasing.

[11] This model has been referred to as the 'benchmark model' of auction theory in the survey article of McAfee and McMillan (1987). See also Matthews (1995), Krishna (2002) and Menezes and Monteiro (2005).

[12] For completeness, if there is more than one bidder with the highest bid, let the object go to each of them with equal probability. The same assumption can be made in the first-price auction. In any case, the probability of a tie will be zero.

$t_i < b^S(v_i) = (v_i + \varepsilon\rho)/(1 + \varepsilon)$.[13] Finally, a similar argument shows that an upward deviation $\tilde{b} > b^S(v_i)$ cannot be profitable. $\square$

Note that $\bar{v}(r) = r + \varepsilon(1 - \lambda)(r - x)$, so that increasing the reserve price can only reduce participation. The reference point effect implies that a participating buyer of type v will bid less than his valuation v (the equilibrium bid in the standard model, where $\varepsilon = 0$) whenever $\rho < v$. In particular, this must be the case if the reference point is solely determined by the reserve price ($\lambda = 1$). Otherwise, the bids can be larger than in the standard model.

Now consider a buyer of type $v \geq \bar{v}(r)$. Let $G(v) = F(v)^{n-1}$ denote the probability that the values of all other buyers are smaller than v. The expected payment of the buyer can then be written as[14]

$$T^S(v) = rG[\bar{v}(r)] + \int_{\bar{v}(r)}^{v} \frac{w + \varepsilon\rho}{1 + \varepsilon} \mathrm{d}G(w).$$

In order to see this, note that he will only win if he has the highest value. He then must pay r if all other buyers have types smaller than $\bar{v}(r)$, and he must pay

$$b^S(w) = \frac{w + \varepsilon\rho}{1 + \varepsilon}$$

if $w \in (\bar{v}(r), v)$ is the highest value of the other $n - 1$ buyers.

1.2. *First-Price Auction*

In a first-price auction, the bidder with the highest bid wins and has to pay what he has bid. As is well known, the bidders do not have dominant strategies in a first-price auction. In the standard model ($\varepsilon = 0$), there is a symmetric equilibrium in which each bidder bids less than his true type. In the present framework, this result can be generalised, so that a bidder who participates in a first price auction bids $b^F(v)$, which is less than $b^S(v)$. More precisely, we get the following result.

PROPOSITION 2 *In a first-price auction, only buyers of type $v \geq \bar{v}(r)$ will participate. Their symmetric equilibrium bidding strategies are given by*

$$b^F(v) = \frac{1}{1 + \varepsilon}\left[v + \varepsilon\rho - \int_{\bar{v}(r)}^{v} \frac{G(w)}{G(v)} \mathrm{d}w\right].$$

Proof. It is obvious that buyer i cannot benefit from participating if $v_i - r - \varepsilon(r - \rho) < 0$. Thus, consider a buyer i with $v_i \geq \bar{v}(r)$. Assume that all other buyers follow the strategy given in the Proposition. Note that $b^F(v)$ is increasing. As a consequence, it is never profitable for buyer i to bid more than $b^F(1)$, because then he would win for sure and could increase his payoff by slightly reducing his bid. Buyer i

[13] Given the tie-breaking rule of the previous footnote, if $\tilde{b} = t_i < b^S(v_i)$, he now loses this positive profit with probability 1/2.

[14] Note that $\bar{v}(r)$ can be negative, while the types are always non-negative. Formally, $F(w)$ and thus $G(w)$ are identical to zero for all $w \leq 0$.

thus considers bidding $b \in [r, b^F(1)]$. Note that there exists a value $z \in [\bar{v}(r), 1]$ such that $b^F(z) = b$. Hence, buyer i's expected payoff from bidding b, which is given by $v_i - b - \varepsilon(b - \rho)$ times the probability that no other buyer bids more than b, can be written as follows:

$$\begin{aligned}
&\{v_i - b^F(z) - \varepsilon[b^F(z) - \rho]\}G(z) \\
&= (v_i + \varepsilon\rho)G(z) - (z + \varepsilon\rho)G(z) + \int_{\bar{v}(r)}^{z} G(w)\mathrm{d}w \\
&= (v_i - z)G(z) + \int_{\bar{v}(r)}^{z} G(w)\mathrm{d}w.
\end{aligned}$$

If buyer i bids $b^F(v_i)$, his expected payoff thus is

$$\int_{\bar{v}(r)}^{v_i} G(w)\mathrm{d}w.$$

Since

$$\begin{aligned}
&(v_i - z)G(z) + \int_{\bar{v}(r)}^{z} G(w)\mathrm{d}w - \int_{\bar{v}(r)}^{v_i} G(w)\mathrm{d}w \\
&= \int_{v_i}^{z} [G(w) - G(z)]\mathrm{d}w \leq 0,
\end{aligned}$$

it cannot be profitable for buyer i to deviate from the strategy given in the proposition. □

Now consider a buyer of type $v \geq \bar{v}(r)$. He pays $b^F(v)$ if all other buyers have types smaller than v, so his expected payment is

$$T^F(v) = b^F(v)G(v) = \frac{1}{1+\varepsilon}\left[(v + \varepsilon\rho)G(v) - \int_{\bar{v}(r)}^{v} G(w)\mathrm{d}w\right].$$

It is easy to check (with integration by parts) that $T^F(v) = T^S(v)$, which is in accordance with the well-known revenue equivalence principle. The winner only pays the second-highest bid in the second-price auction, but the equilibrium bids are lower in the first-price auction, so that the expected payment is the same in both cases.

2. The Optimal Reserve Price

In order to characterise the optimal reserve price, let us now consider the seller's revenue. Recall that the seller does not know the buyers' types. Hence, the seller's expected revenue $\Pi(r, \lambda)$ is simply n times the expected value of the payment that a buyer makes to the seller (which is $T^F(v)$ if $v \geq \bar{v}(r)$, and 0 otherwise).[15] Clearly, the seller will only consider reserve prices such that $\bar{v}(r) < 1$. Then, with integration by parts,

[15] We highlight in our notation the dependence of Π on λ for later purposes (see Section 3 below).

$$\Pi(r,\lambda) = \frac{n}{1+\varepsilon}\int_{\bar{v}(r)}^{1}\left[(v+\varepsilon\rho)G(v) - \int_{\bar{v}(r)}^{v} G(w)\mathrm{d}w\right]\mathrm{d}F(v)$$
$$= \frac{n}{1+\varepsilon}\int_{\bar{v}(r)}^{1}\left[v - \frac{1-F(v)}{f(v)} + \varepsilon\rho\right]F(v)^{n-1}\mathrm{d}F(v).$$

Note that if $\varepsilon = 0$, then the seller's expected revenue is obviously maximised by $r = r_0$, where

$$r_0 - \frac{1-F(r_0)}{f(r_0)} = 0,$$

so that the integrand is positive whenever $v \geq r_0$. The following Proposition characterises the optimal reserve price for $\varepsilon > 0$.

Proposition 3 *The optimal reserve price r^* is given by*

$$r^* = \frac{v^* + \varepsilon(1-\lambda)x}{1+\varepsilon(1-\lambda)},$$

where v^* is uniquely characterised by $\alpha(v^*) = \beta(v^*, n, \varepsilon)$ with

$$\alpha(v) = v - \frac{1-F(v)}{f(v)},$$
$$\beta(v,n,\varepsilon) = \varepsilon\left\{\lambda\frac{1-F(v)^n}{nF(v)^{n-1}f(v)} + (1-\lambda)\left[\frac{1-F(v)}{f(v)} - (1+\varepsilon)x\right] - v\right\}$$

if $\lambda > 0$ or if $\lambda = 0$ and $\varepsilon x \leq 1/f(0)$. If $\lambda = 0$ and $\varepsilon x > 1/f(0)$, then $v^* = 0$.

Proof. The first derivative of the seller's expected profit with respect to r is

$$\frac{\mathrm{d}\Pi(r,\lambda)}{\mathrm{d}r} = \frac{1}{1+\varepsilon}\Bigg(\varepsilon\lambda\{1-F[\bar{v}(r)]^n\} - \left\{\bar{v}(r) - \frac{1-F[\bar{v}(r)]}{f[\bar{v}(r)]} + \varepsilon\rho\right\}$$
$$\times\, nF[\bar{v}(r)]^{n-1}f[\bar{v}(r)][1+\varepsilon(1-\lambda)]\Bigg).$$

The first-order condition can thus be written as

$$\bar{v}(r) - \frac{1-F[\bar{v}(r)]}{f[\bar{v}(r)]} = \varepsilon\lambda\frac{1-F[\bar{v}(r)]^n}{nF[\bar{v}(r)]^{n-1}f[\bar{v}(r)]} + \varepsilon(1-\lambda)\frac{1-F[\bar{v}(r)]}{f[\bar{v}(r)]}$$
$$- \bar{v}(r)\varepsilon(1-\lambda) - \varepsilon\rho[1+\varepsilon(1-\lambda)].$$

The left-hand side is equal to $\alpha[\bar{v}(r)]$, while the right-hand side can be rewritten as $\beta[\bar{v}(r), n, \varepsilon]$.

Consider the case $\lambda > 0$. Note that given the monotone hazard rate assumption, there always exists a unique $v^* > 0$ as defined in the Proposition, because when v moves from zero to one, $\alpha(v)$ strictly increases from $-1/f(0)$ to 1, while $\beta(v, n, \varepsilon)$ strictly decreases from ∞ to $-(1-\lambda)(1+\varepsilon)x\,\varepsilon - \varepsilon < 0$ (the fact that β is decreasing in v follows from the monotone hazard rate property and Lemma 1 in the Appendix). Due to continuity, a straightforward intermediate value argument can thus be applied. The optimal reserve price must be such that $\bar{v}(r) < 1$ and such that $\bar{v}(r) > 0$ (note that $\mathrm{d}\Pi(r, \lambda)/\mathrm{d}r$ is strictly positive if $r \leq \varepsilon(1-\lambda)x/[1+\varepsilon(1-\lambda)]$). Since v^* (and thus r^*, which is implicitly given by $\bar{v}(r^*) = v^*$) is unique and since there must be an interior solution, $\Pi(r, \lambda)$ must attain its maximum at $r = r^*$. Finally, if $\lambda = 0$, it is straightforward to see that there can be a corner solution at $v^* = 0$, which happens if $\varepsilon x > 1/f(0)$. □

We can now analyse the comparative statics properties of our model. In particular, it turns out that the optimal reserve price r^* is increasing in the number of bidders, provided that the reserve price has at least some influence on the reference point. This result is in stark contrast to the standard result, which says that the optimal reserve price r_0 is independent of the number of bidders.[16] In the standard model ($\varepsilon = 0$), the reserve price has only an indirect effect on the seller's expected profit $\Pi(r, \lambda)$, because it merely changes the critical valuation below which a buyer does not participate. In contrast, in our model the reserve price also has a direct positive effect on $\Pi(r, \lambda)$ through the reference point ρ. The more bidders there are, the higher is the expected value of the highest type. Hence, a given reserve price will less likely lead to the no-trade outcome if the number of bidders is increased, so that increasing the reserve price due to the direct reference point effect becomes relatively more attractive.

PROPOSITION 4 *The optimal reserve price r^* is increasing in the number of bidders for all $\lambda > 0$. The optimal reserve price is independent of n if $\lambda = 0$.*

Proof. From the implicit definition of v^* in Proposition 3 it follows that

$$\frac{\mathrm{d}v^*}{\mathrm{d}n} = \frac{\beta_n(v^*, n, \varepsilon)}{\alpha_v(v^*) - \beta_v(v^*, n, \varepsilon)} > 0,$$

where subscripts denote partial derivatives.[17] The denominator is positive, since the monotone hazard rate assumption implies $\alpha_v > 0$, and we already know that $\beta_v < 0$ (see Lemma 1 in the Appendix). Moreover, it is straightforward to check that if $\lambda > 0$, then

$$\beta_n(v^*, n, \varepsilon) = -\varepsilon\lambda \frac{1 - F(v^*)^n + n \ln F(v^*)}{n^2 F(v^*)^{n-1} f(v^*)} > 0,$$

where the inequality follows from the fact that $1 - \xi + \ln \xi < 0$ for $\xi \in (0, 1)$. Hence, if $\lambda > 0$, then v^* is increasing in n and as a consequence,

$$\frac{\mathrm{d}r^*}{\mathrm{d}n} = \frac{1}{1 + \varepsilon(1-\lambda)} \frac{\mathrm{d}v^*}{\mathrm{d}n} > 0.$$

If $\lambda = 0$, then $\beta_n(v^*, n, \varepsilon) = 0$ and thus r^* is independent of n. □

Next, what is the impact of the exogenous reference price x and the weight λ that is associated with the reserve price? An increase in x unambiguously increases a bidder's willingness-to-pay, so that the seller will raise the reserve price, since a larger willingness-to-pay means that the danger of the no-trade outcome is mitigated. An increase in

[16] Notice that our model could be re-interpreted as a modification of the standard model, where the winner must pay $t_i + \varepsilon(t_i - \rho)$ instead of t_i. If the payment $\varepsilon(t_i - \rho)$ accrued to the seller, revenue equivalence between our modified auction and the standard auction would imply that r^* is independent of n. However, in our framework $\varepsilon(t_i - \rho)$ is not paid to the seller, so that her incentives are different.

[17] While the number of bidders is discrete, for simplicity we formally treat n as a continuous variable.

λ can have ambiguous consequences. If x is relatively small, more weight on r means that it becomes more attractive for the seller to increase r^* due to the reference point effect. If x is relatively large and x becomes a less important determinant of the reference point, it may be profitable to reduce r, because *ceteris paribus* the willingness-to-pay of a buyer with a given type will now be smaller, so that the danger of the no-trade outcome becomes more relevant.

PROPOSITION 5 *The optimal reserve price r^* is always increasing in x, but need not be monotone in λ. A sufficient condition for r^* to be increasing in λ is $x \leq v^*$.*

Proof. Consider first the impact of the exogenous reference price x. From the definition of v^*, it follows that

$$\frac{\mathrm{d}v^*}{\mathrm{d}x} = -\frac{\varepsilon(1-\lambda)(1+\varepsilon)}{\alpha_v(v^*) - \beta_v(v^*, n, \varepsilon)} \leq 0$$

(because we already know that the denominator is positive). Since

$$\frac{\mathrm{d}r^*}{\mathrm{d}x} = \frac{\mathrm{d}v^*/\mathrm{d}x + \varepsilon(1-\lambda)}{1+\varepsilon(1-\lambda)},$$

the optimal reserve price is increasing in x if $\alpha_v(v^*) - \beta_v(v^*, n, \varepsilon) > 1 + \varepsilon$, which is always the case because $\alpha_v(v) - \beta_v(v, n, \varepsilon)$ is equal to

$$1 + \varepsilon - \frac{\mathrm{d}}{\mathrm{d}v}\left\{[1 + (1-\lambda)\varepsilon]\frac{1-F(v)}{f(v)} + \varepsilon\lambda\frac{1-F(v)^n}{nF(v)^{n-1}f(v)}\right\},$$

where the derivative of the term in braces is negative.

Next, consider the impact of λ. From the definition of v^*, it follows that

$$\frac{\mathrm{d}v*}{\mathrm{d}\lambda} = \varepsilon\frac{\dfrac{1-F(v)^n}{nF(v)^{n-1}f(v)} - \dfrac{1-F(v)}{f(v)} + (1+\varepsilon)x}{\alpha_v(v^*) - \beta_v(v^*, n, \varepsilon)} > 0,$$

where the inequality holds because

$$\frac{1-F(v)^n}{nF(v)^{n-1}f(v)}$$

is increasing in n. Moreover,

$$\frac{\mathrm{d}r^*}{\mathrm{d}\lambda} = \frac{1}{1+\varepsilon(1-\lambda)}\frac{\mathrm{d}v^*}{\mathrm{d}\lambda} + \varepsilon\frac{v^* - x}{[1+\varepsilon(1-\lambda)]^2},$$

which must be positive if $x \leq v^*$. □

It is also interesting to analyse whether the introduction of a small reference point effect (i.e., a small $\varepsilon > 0$) into the standard model (where $\varepsilon = 0$) reduces or increases the optimal reserve price r^*. In general, r^* will not be monotone in ε.[18] If the reference point

[18] Yet, one can show that the optimal reserve price is monotonically increasing in ε if $\lambda = 1$. To see this, note that in the proof of Proposition 6 we then have $\beta_\varepsilon(v, n, \varepsilon) = [1 - F(v)^n]/[nF(v)^{n-1}f(v)] - v$, which is positive for $v \leq r_0$.

is almost entirely determined by the reserve price (i.e., if λ is close to 1), then making ε slightly positive will lead to r^* being larger than r_0, because of the positive reference point effect. Yet, if λ is close to 0 and x is small, r^* will be smaller than r_0, because the seller predominantly wants to reduce the danger of the no-trade outcome when the preferences become reference-dependent and the (almost exogenous) reference point is low.

PROPOSITION 6 *The introduction of a small reference point effect increases the optimal reserve price, i.e.*

$$\frac{\mathrm{d}r^*}{\mathrm{d}\varepsilon}|_{\varepsilon=0} > 0,$$

if λ is sufficiently large and $n \geq 2$, while it reduces r^* if λ is sufficiently small and $x < r_0$.

Proof. From the implicit definition of v^* in Proposition 3, it follows that

$$\frac{\mathrm{d}v^*}{\mathrm{d}\varepsilon} = \frac{\beta_\varepsilon(v^*, n, \varepsilon)}{\alpha_v(v^*) - \beta_v(v^*, n, \varepsilon)}.$$

We already know that the denominator is positive. At $\varepsilon = 0$, using the definition of r_0, we obtain

$$\beta_\varepsilon(r_0, n, 0) = \lambda\left[\frac{1 - F(r_0)^n}{nF(r_0)^{n-1}f(r_0)} - r_0\right] - (1-\lambda)x.$$

The term in square brackets is positive for $n \geq 2$ (because it is equal to zero for $n = 1$ and increasing in n).[19] Hence, for any given value of x we must have $\beta_\varepsilon > 0$ if λ is sufficiently close to 1. Similarly, $\beta_\varepsilon < 0$ for a given $x > 0$ if λ is sufficiently close to 0. From Proposition 3, it also follows that

$$\left.\frac{\mathrm{d}r^*}{\mathrm{d}\varepsilon}\right|_{\varepsilon=0} = \left.\frac{\mathrm{d}v^*}{\mathrm{d}\varepsilon}\right|_{\varepsilon=0} + (1-\lambda)(x - r_0).$$

Hence, the introduction of a small reference point effect into the standard model will increase the optimal reserve price if λ is sufficiently close to 1, while it will decrease the optimal reserve price if λ is sufficiently close to 0 and $x < r_0$. □

Even though we think that small values of ε are most plausible, in order to understand the model better it is interesting to note what happens if ε becomes large. Suppose that $\lambda < 1$. When ε goes to infinity, r^* must converge to x, because in the limit the buyers are basically unwilling to pay more than ρ (which rules out a minimum bid r larger than ρ) and the seller can increase her profit by increasing r if it is smaller than ρ. If $\lambda = 1$, in the limit a buyer will never be willing to pay more than r, so the optimal reserve price converges to the price posted by a profit-maximising seller who can only use a fixed-price mechanism. Such a seller will set a price p in order to maximise her expected profit $p[1 - F(p)^n]$, which is the price times the probability that there is at least one buyer willing to pay the price. The first-order condition is $1 - F(p^*)^n - p^* nF(p^*)^{n-1}f(p^*) = 0$. Formally, when ε goes to infinity, inspection of

[19] Note that in the special case $n = 1$ and $\lambda = 1$, the optimal reserve price does not depend on ε. In this case, $\rho = r$ and the bidder has to pay r if he gets the object, so that we are back in the standard model regardless of ε.

Proposition 3 immediately reveals that v^* must converge to zero (so that r^* converges to x) if $\lambda < 1$, and $v^* = r^*$ converges to p^* if $\lambda = 1$.

Remark 1 *It should be emphasised that even if* ε *is very small, the impact of the reference point effect on the optimal reserve price* r^* *can be significant if the number of potential buyers is sufficiently large. For example, consider the uniform distribution, so that* $r_0 = 0.5$. *Assume that* $\varepsilon = 0.01$; *i.e., the reference point effect is quite small. If* $\lambda = 1$, *the optimal reserve price for* $n = 3$ *is only slightly increased to* $r^* \approx 0.503$, *but for* $n = 30$ *it is significantly increased to* $r^* \approx 0.774$.

Finally, note that our results are relevant even if the reserve price has only a very small influence on what the buyers perceive as a reference point. As an illustration, let $\lambda = 0.01$ in the example mentioned in Remark 1. The optimal reserve price in the case of thirty bidders is now between $r^* \approx 0.665$ (if $x = 0$) and $r^* \approx 0.674$ (if $x = 1$), which is still quite different from the standard result r_0.

3. Applications

3.1. *Secret Reserve Prices*

In the previous Section we have characterised the optimal reserve price r^* under the assumption that the seller publicly announces the reserve price. Now consider what happens if the seller keeps the reserve price secret. In this case, it can no longer influence the bidders' reference point, which is now entirely determined by the exogenous reference price x. Note that in a *second-price auction,* it is still a dominant strategy for a buyer of type v to bid

$$b^S(v) = \frac{v + \varepsilon x}{1 + \varepsilon},$$

so that the seller's expected revenue is now given by $\Pi(r, 0)$. Hence, the optimal reserve price can be derived as in the previous section. In stark contrast to the standard model (Riley and Samuelson, 1981), it turns out that the seller's expected revenue may well be larger if she keeps the reserve price secret. In order to see this, note that due to the envelope theorem

$$\begin{aligned}\frac{d\Pi(r^*,\lambda)}{d\lambda} &= \frac{n}{1+\varepsilon}\Bigg(\int_{\bar{v}(r^*)}^{1} \varepsilon(r^* - x)F(v)^{n-1}\mathrm{d}F(v) + \Bigg\{\bar{v}(r^*) - \frac{1 - F[\bar{v}(r^*)]}{f[\bar{v}(r^*)]} \\ &\quad + \varepsilon[\lambda r^* + (1-\lambda)x]\Bigg\}F[\bar{v}(r^*)]^{n-1}f[\bar{v}(r^*)]\varepsilon(r^* - x)\Bigg) \\ &= \frac{\varepsilon(r^* - x)}{1+\varepsilon}\Bigg(1 - F(v^*)^n + \Bigg\{v^* - \frac{1 - F(v^*)}{f(v^*)} \\ &\quad + \varepsilon[\lambda r^* + (1-\lambda)x]\Bigg\}nF(v^*)^{n-1}f(v^*)\Bigg).\end{aligned}$$

Hence, $\Pi(r^*, \lambda)$ is decreasing in λ whenever[20]

[20] Note that the expression in braces is positive, because $[1 - F(v^*)^n]/[nF(v^*)^{n-1}f(v^*)]$ is increasing in n (see the proof of Proposition 4).

$$(r^* - x)\left\{\frac{1 - F(v^*)^n}{nF(v^*)^{n-1}f(v^*)} - \frac{1 - F(v^*)}{f(v^*)} + v^* + \varepsilon[\lambda r^* + (1-\lambda)x]\right\} < 0,$$

which is the case whenever $x > r^*$, or equivalently $x > v^*$. If $\Pi(r^*, \lambda)$ is decreasing in λ, the seller's expected profit attains its maximum at $\lambda = 0$ (which corresponds to the case of a secret reserve price). Thus, if this condition is satisfied,[21] then the seller is strictly better off if she keeps the reserve price secret, regardless of the weight $\lambda > 0$ with which a publicly announced reserve price enters the bidders' reference point.

If the reserve price is kept secret, first-price auctions and second-price auctions are no longer revenue-equivalent. Note that in a *first-price auction* the bids depend on the reserve price (see Proposition 2). Consider now the case of a secret reserve price, so that the reference point is entirely determined by x. If a bidder of type v believes that the seller has set the reserve price $\hat{r}$, then in a first-price auction he now bids

$$\hat{b}^F(v) = \frac{1}{1+\varepsilon}\left[v + \varepsilon x - \int_{\bar{v}(\hat{r})}^{v} \frac{G(w)}{G(v)}\mathrm{d}w\right].$$

Consider a bidder of type v whose bid lies above r. From the seller's point of view, the expected payment of the bidder is $\hat{T}^F(v) = \hat{b}^F(v)G(v)$, which (in contrast to $T^S(v)$, the payment she would expect from the bidder in a second-price auction) does not depend on the level of r actually set by the seller. Let $\hat{v}(r)$ be such that $\hat{b}^F[\hat{v}(r)] = r$. The seller's expected profit then is

$$n\int_{\hat{v}(r)}^{1} \hat{b}^F(v)G(v)\mathrm{d}F(v),$$

which is obviously maximised if $\hat{v}(r) = 0$. In equilibrium the condition $\hat{r} = r$ must be satisfied, so the seller's expected profit in a first-price auction with a secret reserve price is given by $\Pi[\varepsilon x/(1+\varepsilon), 0]$. Since in general the reserve price thus is not the one that maximises $\Pi(r, 0)$, it follows that with secret reserve prices the expected revenue in a first-price auction is smaller than in a second-price auction.[22]

Moreover, even in a first-price auction it is possible that the seller's expected revenue with a secret reserve price is larger than the one she could attain by publicly announcing r^*. Even though it is no longer sufficient that $\Pi(r^*, \lambda)$ is decreasing in λ, the positive impact of the larger weight on x can overcompensate the negative impact of the sub-optimal secret reserve price, if the exogenous reference price x is large. As an illustration, Figure 1 shows the seller's expected profits with public and secret reserve prices as functions of x if the valuations are uniformly distributed, $n = 2$, $\varepsilon = 0.3$, and $\lambda = 0.5$.

Finally, it should be noted that there are alternative ways in which one could introduce secret reserve prices into our framework. For example, one might imagine that in the presence of a secret reserve price, the reference point could be less relevant (i.e., ε could be smaller if r is not made public). Moreover, a bidder might try to guess

[21] Since v^* is increasing in λ (see the proof of Proposition 5), a sufficient condition for $x > v^*$ is that x is larger than v^* at $\lambda = 1$. For example, in the case of the uniform distribution and $n = 2$, this condition reads $x > (1 + \sqrt{1 + 4\varepsilon + 3\varepsilon^2})/(4 + 3\varepsilon)$, which is always the case if $x > \sqrt{3}/3$.

[22] If $\varepsilon x < 1/f(0)$, the expected revenue in a first-price auction is strictly smaller. Otherwise, the expected revenues in first-price and second-price auctions are identical (cf. Proposition 3).

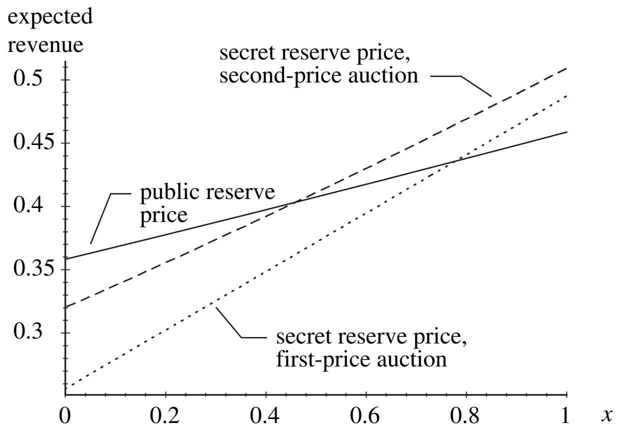

Fig. 1. *Secret Reserve Prices*

the secret reserve price, and his belief might affect his reference point (but the weight might be smaller than λ). Further experimental research could be very helpful in order to find the most suitable modelling alternative.

3.2. *The Value of Additional Bidders*

In the standard auction model with $\varepsilon = 0$, Bulow and Klemperer (1996) have shown that the expected profit of a seller who sets an optimal reserve price in the presence of n potential buyers is smaller than the expected profit of a seller who cannot set a reserve price when there are $n + 1$ potential buyers. Hence, even if n is large, the marginal value of one additional bidder is greater than the benefit of setting an optimal reserve price. This striking result is no longer true if there is a reference point effect ($\varepsilon > 0$), given that the number of bidders is sufficiently large. In order to see this, note that the seller's profit in the absence of a reserve price (i.e., if ρ is entirely determined by x) can be written as (with integration by parts)

$$\begin{aligned}\Pi(0,0) &= \frac{n}{1+\varepsilon}\int_0^1 \left[v - \frac{1-F(v)}{f(v)} + \varepsilon x\right] F(v)^{n-1}\mathrm{d}F(v)\\ &= \frac{n}{1+\varepsilon}\int_0^1 \{vf(v) - [1-F(v)] + \varepsilon x f(v)\}F(v)^{n-1}\mathrm{d}v\\ &= \frac{n}{1+\varepsilon}\int_0^1 (v+\varepsilon x)[1-F(v)](n-1)F(v)^{n-2}f(v)\mathrm{d}v\\ &= \frac{1}{1+\varepsilon}\{\mathrm{E}[\tilde{v}_{(2)}] + \varepsilon x\}\end{aligned}$$

where $\tilde{v}_{(2)}$ is the second highest element of $\{\tilde{v}_1, \ldots, \tilde{v}_n\}$.[23] Notice that $\Pi(0, 0)$ is increasing in n and it converges to $(1 + \varepsilon x)/(1 + \varepsilon) < 1$ if n goes to infinity, for any

[23] Thus, the distribution function of $\tilde{v}_{(2)}$ is $\Pr\{\tilde{v}_{(2)} \leq v\} = F(v)^n + nF(v)^{n-1}[1 - F(v)]$, and the density function is $n(n-1)F(v)^{n-2}f(v)[1 - F(v)]$.

given $\varepsilon > 0$ and $x < 1$. It is straightforward to see that $\Pi(r^*, \lambda)$ must increase in n and converge to 1 (this would even be the case if the seller could only post a fixed price). Hence, if n is sufficiently large and $x < 1$, the value of an additional buyer will be smaller than the benefit from setting the optimal reserve price. In contrast to the standard model, the present analysis can thus explain why restricting the number of bidders – when this makes commitment to a reserve price more credible, as has been supposed by Bulow and Klemperer (1996) – can indeed be profitable for the seller.[24]

4. Concluding Remarks

We have shown that prominent results of the, by now, standard private independent values model of auctions with symmetric bidders are not robust when the bidders' utilities are influenced by a (possibly very small) reference point effect. Optimal reserve prices may be quite different from what standard theory prescribes,[25] they may well increase with the number of bidders, and keeping them secret can be profitable. The value of additional bidders might be smaller than has previously been thought.

It could be an interesting avenue for future research to analyse other selling mechanisms when utilities are reference-dependent. In the standard model, the revelation principle allows us to maximise over the (infinitely large) class of all conceivable mechanisms, because every mechanism is equivalent to a suitably chosen direct revelation mechanism. Hence, optimal mechanisms can be characterised that cannot be improved upon. It is questionable whether this approach is convincing from a behavioural economics perspective.[26] In particular, we think that in our context the relevant reference point in general cannot be independent of the auction format. For example, while in the standard model an optimal reserve price is equivalent to an optimal entry fee, this does not need to be the case in our model. Future experimental studies might help to find out how the weights attached to reserve prices and entry fees differ. Similarly, the starting price of a Dutch auction might influence the reference point. In particular, the reference point might be adapted during the oral bidding process in an open auction format, which may well help to explain the phenomenon known as bidding fever, which is a puzzle for the standard analysis. Moreover, in sequential auctions it might be a good idea to start with more expensive goods, because the price obtained in period t might influence the reference point in period $t + 1$.

It could also be an interesting topic for future research to incorporate other insights from behavioural economics into auction theory. For example, the endowment effect,

[24] Of course, the finding of Bulow and Klemperer (1996) is valid if the number of potential buyers and the reference point effect are sufficiently small. For example, let v be uniformly distributed, $x = 0.1$, and $\lambda = 0.9$. Then the expected profit with the optimal reserve price and $n = 2$ is smaller than the expected profit with no reserve price and $n = 3$ if $\varepsilon < \hat{\varepsilon}$, with $\hat{\varepsilon} \approx 0.35$.

[25] Note that increasing reserve prices over and above the usually prescribed levels has also been suggested in the literature on bidding rings, see Graham and Marshall (1987) and Mailath and Zemsky (1991).

[26] Indeed, many experiments have shown that framing effects are highly important. Hence, what is equivalent in traditional economic theory does not need to be equivalent in the view of real people. See Tversky and Kahneman (1981) and Kahneman and Tversky (1984) with regard to framing and also Masatlioglu and Uler (2004), who show a related point in an auction experiment.

according to which ownership of an object appears to increase one's valuation, might have an interesting impact on auction models with resale opportunities.[27] In our view, exploring the implications of departures from standard economic paradigms seems to be an exciting and promising task for auction theorists.

Appendix

LEMMA 1 *The term*

$$\frac{1 - F(v)^n}{nF(v)^{n-1}f(v)}$$

is strictly decreasing in v *for all* $v \in (0, 1)$.

Proof. We want to show that

$$\frac{\mathrm{d}}{\mathrm{d}v}\left[\frac{1 - F(v)^n}{F(v)^{n-1}f(v)}\right] = \frac{1}{[F(v)^{n-1}f(v)]^2}\Big\{-nF(v)^{2n-2}f(v)^2 - [1 - F(v)^n][(n-1)F(v)^{n-2}f(v)^2 + F(v)^{n-1}f'(v)]\Big\}$$

is negative. In order to see that this is indeed the case, multiply with

$$[F(v)^{n-1}f(v)]^2\frac{1 - F(v)}{[1 - F(v)^n]F(v)^{n-1}} > 0,$$

so that we must show

$$\left\{-\frac{nF(v)^{n-1}[1 - F(v)]}{1 - F(v)^n} - (n-1)\frac{1 - F(v)}{F(v)}\right\}f(v)^2 - [1 - F(v)]f'(v) < 0.$$

Since $f(v)^2 + [1 - F(v)]f'(v) > 0$ due to the monotone hazard rate assumption, the left-hand side is smaller than

$$\begin{aligned}&\left\{-\frac{nF(v)^{n-1}[1 - F(v)]}{1 - F(v)^n} - (n-1)\frac{1 - F(v)}{F(v)} + 1\right\}f(v)^2\\ &= \frac{[1 - F(v)]\{-nF(v)^n - (n-1)[1 - F(v)^n]\} + [1 - F(v)^n]F(v)}{[1 - F(v)^n]F(v)}f(v)^2\\ &= \{1 - F(v)^n - n[1 - F(v)]\}\frac{f(v)^2}{[1 - F(v)^n]F(v)} \leq 0,\end{aligned}$$

where the inequality follows from the fact that the term in braces is (increasing in v and thus) always smaller than $1 - F(1)^n - n[1 - F(1)] = 0$.

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[27] For instance, Zheng (2002) has shown that the optimal allocation derived by Myerson (1981) can under certain circumstances also be achieved when the bidders cannot commit not to resell. Yet, this requires resale to take place, which is less probable to happen if there is an endowment effect. But if the endowment effect is sufficiently strong, we are again in the world of Myerson (1981). Hence, small endowment effects might be the most damaging ones from the seller's viewpoint.

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