*The Economic Journal*, Doi: 10.1111/j.1468-0297.2010.02405.x. 
Published by Blackwell Publishing, 9600 Garsington Road, Oxford OX4 2DQ, UK and 350 Main Street, Malden, MA 02148, USA.

# Technical Appendix to

# UNIONISATION TRIGGERS TAX INCENTIVES TO ATTRACT FOREIGN DIRECT INVESTMENT

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ECONOMIC JOURNAL, doi: 10.1111/j.1468-0297.2010.02405.x

## Appendices

### A. *Proof of Lemma 1*

We start from country $A$'s indirect utility in Regime $A$, $V_A^A$, as given in $(20a)$. In Regime $A2$, where the union's wage is not constrained by the condition to attract the outside firm, we substitute $\tilde{w}^A$ from (17) into $(20a)$. This gives

$$V_A^{A2} = \frac{(2n+7)(\alpha - \bar{w})^2}{36\beta} + n\bar{w} + t_A + \sigma.$$

Hence, $\partial V_A^{A2}/\partial t_A = 1$ holds throughout Regime $A2$, implying that it is optimal for country $A$'s government to raise taxes until Regime $A1$ is reached.

In Regime $A1$, we substitute $w_{\max}^A$ from (13) into $(20a)$. This gives

$$V_A^{A1} = \frac{\delta^2(2n-5) + 24\delta(\alpha - \bar{w})}{144\beta} + n\bar{w} + t_A + \sigma, \tag{A.1}$$

where $\delta(t_A, t_B)$ is given in (13). Maximising with respect to $t_A$, for any given level of $t_B$, yields

$$t_A^m = \frac{(16-5n)(5n-4)(\alpha - \bar{w})^2}{144(2-n)^2\beta} + t_B. \tag{A.2}$$

The second-order condition for a maximum is fulfilled, as

$$\frac{\partial^2 V_A^{A1}}{\partial t_A^2} = -864\beta(\alpha - \bar{w})\delta^{-3/2} < 0. \tag{A.3}$$

Hence, country $A$ either sets $t_A^m$ in (A.2), or it raises its tax until it reaches the border to Regime $B$, where $(t_A - t_B)^H$ holds as given in (16). For any given $t_B$, country $A$ thus wants to raise its tax throughout Regime $A1$ iff $t_A^m > t_A^H$. This condition is fulfilled if

$$\frac{(16-5n)(5n-4)(\alpha - \bar{w})^2}{144(n-2)^2\beta} - \frac{\left(2\sqrt{3}-9\right)(\alpha - \bar{w})^2}{72\beta} > 0 \quad \Leftrightarrow \quad n > 2\left(3\sqrt{3}-5\right) \approx 0.39. \tag{A.4}$$

Hence, for any $n \geq 0.4$ the tax differential in any candidate equilibrium in Regime $A$ is given by $(t_A - t_B)^H$ in (16).

## B. *Proof of Proposition 1*

We first show that country $A$ cannot gain from deviating from $\tilde{t}_A$ when country $B$ sets $t_B = t_B^o$. First, consider $t_A < \tilde{t}_A$, which leads to a tax pair in the interior of Regime *A1* (19). Differentiating (A.1) with respect to $t_A$ and re-substituting $\delta$ from (13) gives

$$\frac{\partial V_A^{A1}}{\partial t_A} = 2(n-2) + \frac{12(\alpha - \bar{w})}{\delta}. \tag{B.1}$$

Evaluating (B.1) at (30) and using $n \geq 0.4$ from Lemma 1 shows that $\partial V_A^{A1}/\partial t_A > 0$ at $(\tilde{t}_A, t_B^o)$. Hence, reducing $t_A$ below $\tilde{t}_A$ would be welfare-decreasing for country $A$.

If instead $t_A > \tilde{t}_A$ then, from the construction of $\tilde{t}_A$, we get $(t_A - t_B) > (t_A - t_B)^H$. Hence, from (19) the union of country $A$ will induce an allocation in Regime $B$. Substituting $\tilde{t}_A$ from (29) into $V_A^A$ (23*a*) and comparing with $V_A^B$ (23*b*) gives

$$V_A^A - V_A^B = \frac{(32\sqrt{3} - 51)(\alpha - \bar{w})}{288\beta} > 0. \tag{B.2}$$

Hence, welfare in country $A$ falls if $t_A > \tilde{t}_A$, implying that $\tilde{t}_A$ is country $A$'s optimal tax choice if $t_B = t_B^o$.

Next, we ask whether country $B$ can gain from deviating from $t_B^o$ when $t_A$ is set at $\tilde{t}_A$. If country $B$ sets $t_B < t_B^o$ then $(t_A - t_B) > (t_A - t_B)^H$, resulting in an allocation in Regime $B$. By construction, $t_B^o$ is the minimum tax rate that country $B$ needs to levy in order to be equally well off in Regime $B$ as compared to Regime $A$, that is $V_B^B(t_B^o) = V_B^A$. As $V_B^B$ is rising in $t_B$ (cf. (23*b*)), it must be true that $V_B^B(t_B < t_B^o) < V_B^A$ and hence country $B$ is worse off if it chooses $t_B < t_B^o$.

Finally, we consider $t_B > t_B^o$. In this case the allocation remains in Regime *A1* and there are no direct effects on welfare in country $B$. However, from (13) the wage rate $w_{\max}^A$ rises as the union in country $A$ exploits its increased wage setting power, reducing $V_B^A$ from (20*a*). Thus country $B$ will be worse off by raising $t_B$ above $t_B^o$. With neither country having an incentive to deviate from its tax policy, taking into account the adjustments in later stages of the game, the tax pair $(\tilde{t}_A, t_B^o)$ represents a subgame-perfect Nash equilibrium of the first-stage game.

## C. *Proof of Proposition 2*

From equation (8), the outside firm's profits in country $A$ are $(\alpha - w^A)^2/9\beta$, and the profits in country $B$ are $(\alpha - 2\bar{w} + w^B)^2/9\beta$. From the maximisation of the trade union (10), $w^B > \bar{w}$, which implies from the trade union's arbitrage condition that $w^A > \bar{w}$ must also hold. Hence, $\pi_c^B > \pi_c^A$, implying that the outside firm always locates in country $B$ when taxes are exogenously constrained to zero.

In Regime $B$, welfare in country $A$ is given by $V_A^B$ in (23*b*). In the tax competition equilibrium in Regime $A$ country $A$'s welfare is given by $V_A^A$ in (23*a*), where the equilibrium tax rate $\tilde{t}_A$ from (30) must be substituted. This yields a welfare gain for country $A$ from the tax competition game, which is given in (B.2).

For country $B$ welfare in the absence of taxes is given by $V_B^B$ in (23*b*) where $t_B$ is set to zero. In the tax competition equilibrium in Regime $A$ country $B$'s welfare is given by $V_B^A$ in (23*a*). The welfare difference for country $B$ is thus:

$$V_B^A - V_B^B = \frac{(1-n)(\alpha - \bar{w})^2(16\sqrt{3} - 21)}{288\beta} - \sigma, \tag{C.1}$$

which is positive only if $\sigma$ is not too large.

Finally, firm $c$'s profits in the absence of taxes are given by $\pi_c^B = 25(\alpha - \bar{w})^2/144\beta$. The after-tax profits in the tax competition game are $\pi_c^A - \tilde{t}_A$. Substituting in from (21) and (30) and forming the difference yields

$$\pi_c^A - \tilde{t}_a - \pi_c^B = \frac{-(1-n)(16\sqrt{3}-21)(\alpha-\bar{w})^2}{288\beta} + \sigma, \tag{C.2}$$

which is negative if $\sigma$ is not too large. Adding up (B.2)–(C.2) shows that the the change in global welfare from the tax competition game, in comparison to a scenario without taxes, equals the isolated welfare change in country $A$:

$$V^A - V^B \equiv V_A^A + V_B^A + \pi_c^A - \tilde{t}_A - (V_A^B + V_B^B + \pi_c^B) = \frac{(32\sqrt{3}-51)(\alpha-\bar{w})}{288\beta} > 0.$$

This demonstrates that global welfare is higher in the tax competition equilibrium, as a result of firm $c$ locating in country $A$.

## D. *Proof of Proposition 3*

From (20$a$) country $A$'s indirect utility, net of firm profits (superscript $w$), in Regime $A$ is

$$(V_A^A)^w = \frac{2n(\alpha-w^A)^2}{9\beta} + \frac{2(w^A-\bar{w})(\alpha-w^A)}{3\beta} + n\bar{w} + t_A + \sigma.$$

We substitute $(w^A)^*$ from (21) and $\tilde{t}_A$ from (30). To obtain the per capita welfare of a *unionised* worker, we divide tax revenues and consumer surplus by $n$ but the wage surplus by $sn$, as only the share of workers in sector $x$ enjoys the wage surplus. This gives

$$v_A^{\text{union}} = \left[\frac{7+4\sqrt{3}}{72} + \frac{1}{24ns} + \frac{(21-16\sqrt{3})n + 24\sqrt{3} - 57}{288n}\right]\frac{(\alpha-\bar{w})^2}{\beta} + \bar{w}. \tag{D.1}$$

In the absence of a trade union, both countries are indifferent about attracting the firm, except for the technological externality $\sigma$. This leads to $t_i = -\sigma$ in equilibrium and both countries are indifferent about being in Regime $A$ or $B$. Hence per capita welfare (without profit income) of all workers in country $A$ amounts to

$$v_A^{\text{nounion}} = \frac{2(\alpha-\bar{w})^2}{9\beta} + \bar{w}. \tag{D.2}$$

Equating per capita welfare in (D.1) and (D.2) and solving for $s$ yields the critical share $s^c$ where unionised workers are indifferent between having the union or not:

$$s^c = \frac{4}{19 - 8\sqrt{3} + 5n}. \tag{D.3}$$

This share is falling in $n$ and thus reaches its minimum at the maximum value of $n$, which is unity. In this case $s^c(n=1) \approx 0.394$. Hence, $s < 0.394$ is a sufficient condition for each unionised worker to gain from having the union.

## E. *The Valuation of Profit Income in Country A*

When profit income in country $A$ is valued with the exogenous factor $\gamma$, its welfare expressions in (20$a$) and (20$b$) change to:

$$V_A^A = \frac{2n(\alpha-w^A)^2}{9\beta} + \frac{\gamma(\alpha-w^A)^2}{9\beta} + \frac{2(w^A-\bar{w})(\alpha-w^A)}{3\beta} + n\bar{w} + t_A + \sigma, \tag{E.1}$$

$$V_A^B = \frac{n(2\alpha-\bar{w}-w^B)^2}{18\beta} + \frac{\gamma(\alpha-2w^B+\bar{w})^2}{9\beta} + \frac{(\alpha-\bar{w})^2}{24\beta} + n\bar{w}. \tag{E.2}$$

With this extension, the condition that ensures Lemma 1 holds (see Appendix A) generalises to:

$$144-(7+4\sqrt{3})(2n+\gamma-5)^2>0. \tag{E.3}$$

Condition (E.3) is the more likely to hold, the larger is country $A$'s population size $n$ and the larger is the valuation $\gamma$ of firm $a$'s profits. Focusing on combinations of $n$ and $\gamma$ for which (E.3) is fulfilled, we can substitute $(w^A)^*$ from (21) and $w^B$ from (10) into (E.1) and (E.2), respectively. This gives for country $A$'s welfare in Regime $A$:

$$V_A^A=\frac{[6+(2n+\gamma)(7+4\sqrt{3})](\alpha-\bar{w})^2}{144\beta}+n\bar{w}+t_A+\sigma$$

and analogously in Regime $B$:

$$V_A^B=\frac{(12+49n+8\gamma)(\alpha-\bar{w})^2}{288\beta}+n\bar{w}.$$

Setting $V_A^A = V_A^B$ gives a changed value for country $A$'s best offer:

$$t_A^o=\frac{-[(6+8\sqrt{3})\gamma+n(16\sqrt{3}-21)](\alpha-\bar{w})^2}{288\beta}-\sigma. \tag{E.4}$$

In contrast, $t_B^o$ and hence $\tilde{t}_A$ are unchanged from (29). Comparing (E.4) and (29) gives the following condition for $\tilde{t}_A > t_A^o$, and hence for the tax competition equilibrium to lie in Regime $A$:

$$\tilde{t}_A>t_A^o \quad \text{iff} \quad \gamma>\gamma^c\approx 0.78. \tag{E.5}$$

F. *The Model with Positive Trade Costs*

We assume that there is a per unit trade cost of $\tau$ on each unit of good $x$ shipped between countries $A$ and $B$, whereas trade in the $z$ industry remains free. Assuming segmented markets, so that firms can maximise profits in each market independently, profits in the final stage of the game are

$$\pi_a^A=\pi_c^A=\frac{n(\alpha-w^A)^2}{9\beta}+\frac{(1-n)(\alpha-\tau-w^A)^2}{9\beta}$$

if the outside firm $c$ goes to $A$. If it goes to $B$ then

$$\begin{aligned}\pi_a^B&=\frac{n(\alpha+\bar{w}+\tau-2w^B)^2}{9\beta}+\frac{(1-n)(\alpha+\bar{w}-2\tau-2w^B)^2}{9\beta},\\ \pi_c^B&=\frac{n(\alpha-2\bar{w}-2\tau+w^B)^2}{9\beta}+\frac{(1-n)(\alpha-2\bar{w}+\tau+w^B)^2}{9\beta}.\end{aligned} \tag{F.1}$$

The quantities produced in the two regimes are

$$x_a^A=x_c^A=\frac{n(\alpha-w^A)}{3\beta}+\frac{(1-n)(\alpha-\tau-w^A)}{3\beta}$$

and

$$\begin{aligned}x_a^B&=\frac{n(\alpha+\bar{w}+\tau-2w^B)}{3\beta}+\frac{(1-n)(\alpha+\bar{w}-2\tau-2w^B)}{3\beta},\\ x_c^B&=\frac{n(\alpha-2\bar{w}-2\tau+w^B)}{3\beta}+\frac{(1-n)(\alpha-2\bar{w}+\tau+w^B)}{3\beta},\end{aligned} \tag{F.2}$$

respectively. In stage 4, using these quantities in (9) and maximising with respect to $w$ in Regime $B$ gives

$$w^B = \frac{1}{4}[\alpha + 3\bar{w} + (3n-2)\tau], \qquad \Omega^B = \frac{[\alpha - \bar{w} + (3n-2)\tau]^2}{24\beta} \tag{F.3}$$

as the union's wage and the wage surplus in Regime $B$. In stage 3, equating the multinational's gross profit differential in the two countries to $(t_A - t_B)$, as in (12), solving this term for $w^A$ and substituting $w^B$ from (F.3) gives

$$\begin{aligned} w^A = {} & \alpha - (1-n)\tau \\ & - \frac{1}{4}\sqrt{25(\alpha-\bar{w})^2 - 10(9n-2)\tau(\alpha-\bar{w}) + (2-n)(47n+2)\tau^2 + 144\beta(t_A - t_B)}, \end{aligned} \tag{F.4}$$

which collapses to (13) for $\tau = 0$. In stage 2, we use (F.4) and (F.2) to calculate the wage surplus $\Omega^A$. Equating this to $\Omega^B$ in (F.3) yields

$$\begin{aligned} (t_A - t_B)^H = {} & \tfrac{1}{72\beta}[-9(\alpha-\bar{w})^2 + 2(25n-8)\tau(\alpha-\bar{w}) + n(23n-48)\tau^2 \\ & + 2\sqrt{[3\alpha - 3\bar{w} - (4-5n)\tau](\alpha - \bar{w} - n\tau)(\alpha - \bar{w} + n\tau - \tau)^2}], \end{aligned} \tag{F.5}$$

which reduces to (16) for $\tau = 0$.

In the first stage of the game, the governments maximise welfare in a way analogous to (20$a$) and (20$b$). The firms' profits are given in (F.1) and the wage surplus of country $A$'s union is obtained by inserting (F.2) into (9) and (14). Consumer surplus in the different regimes amounts to

$$\begin{aligned} CS_A^A &= \frac{2n(\alpha - w^A)^2}{9\beta}, \qquad & CS_B^A &= \frac{2(1-n)(\alpha - w^A - \tau)^2}{9\beta}, \\ CS_A^B &= \frac{n(2\alpha - \bar{w} - \tau - w^B)^2}{18\beta}, \qquad & CS_B^B &= \frac{(1-n)(2\alpha - \bar{w} - \tau - w^B)^2}{18\beta}. \end{aligned} \tag{F.6}$$

Substituting (F.3), (F.4) and (F.5) into the welfare terms and equalising country $B$'s welfare in the two regimes, we derive country $B$'s best offer tax rate as

$$\begin{aligned} t_B^o = {} & -\sigma + \frac{1}{288\beta}\Big\{-[7\alpha - 7\bar{w} - (3n+2)\tau]^2 \\ & + 4\Big[\sqrt{7(\alpha-\bar{w})^2 + 2(5n-6)\tau(\alpha-\bar{w}) - (n^2+4n-4)\tau^2 + 4\mu(\alpha - \bar{w} + n\tau - \tau)} - 4n\tau\Big]^2\Big\} \end{aligned} \tag{F.7}$$

where

$$\mu \equiv \sqrt{3(\alpha-\bar{w})^2 - 2(2-n)\tau(\alpha-\bar{w}) + (4-5n)n\tau^2}.$$

It is straightforward to show that Lemma 1 also holds for $n \geq 0.4$ when $\tau > 0$. Intuitively this is because the presence of trade costs makes it less attractive for country $A$ to pursue a beggar-thy-neighbour policy by means of high unionised wages. Hence, we can add the tax differential $(t_A - t_B)^H$ from (F.5) to (F.7) to derive the tax rate that country $A$ will optimally offer in a candidate Regime $A$ equilibrium. Substituting this tax rate to get maximised Regime $A$ welfare, $(V_A^A)^*$, and subtracting $V_A^B$ determines under which conditions country $A$ wants to host the firm.This difference is

$$(V_A^A)^* - V_A^B = \frac{1}{288\beta}\Big\{-51(\alpha - \bar{w})^2 + 2(127n - 38)(\alpha - \bar{w})\tau + [n(197n - 324) - 12]\tau^2 + 32\mu[\alpha - \bar{w} + (n-1)\tau]\Big\}, \tag{F.8}$$

where $\mu$ is defined above. Setting this welfare difference equal to zero yields the locus of all $(n, \tau)$ combinations where country $A$ is indifferent about attracting the firm or not. This is the upward sloping curve labelled $(V_A^A)^* = V_A^B$ in Figure 3. Below this line (F.8) is positive and a Regime $A$ equilibrium results, whereas above the line (F.8) is negative and the equilibrium is in Regime $B$.

To obtain the lowest prohibitive level of trade costs where firm $a$ stops exporting to country $B$, we substitute the equilibrium wages (F.3) and (F.4) into the profit terms (F.1) and set profits equal to zero. This yields a lowest prohibitive trade cost level of $\bar{\tau} = (\alpha - \bar{w})/(3n + 2)$, which arises in Regime $B$. This upper limit on $\tau$ is represented by the downward sloping curve in Figure 3.