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# Technical Appendix to

# THE OPTIMAL LEVEL OF INTERNATIONAL RESERVES FOR EMERGING MARKET COUNTRIES: A NEW FORMULA AND SOME APPLICATIONS

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## Appendix

Table A1

*List of Variables Used in Regression Analysis*

| Variable | Source |
|---|---|
| Debt | |
| Lag of real public debt to real GDP | GDF/WDI (2006) |
| Lag of short-term debt to real GDP | GDF/WDI (2006) |
| Stock of reserves | |
| Total reserves minus gold (line 11.d) GDP | IFS (2006) |
| Balance of payments | |
| Current account (line 78ald) | IFS (2006) |
| Reserves and related assets (line 79dad) | IFS (2006) |
| Exchange rate | |
| Second lag exchange rate regime dummies | Reinhart and Rogoff (2004) |
| Lag of Real Effective Exchange Rate Deviation from HP trend | |
| Trade | |
| Lag of openness to trade | WDI (2006) |
| Lag of term of trade growth | IFS (2006) |
| Index of current account openness | Quinn and Toyoda (2008) |
| US interest rate | |
| Interest rate of T-bill | IFS (2006) |
| Change in the interest rate of T-bill | IFS (2006) |
| Financial development | |
| Stock market capitalisation over GDP | Beck and Levine (2005) |
| Stock market total value traded over GDP | Beck and Levine (2005) |
| Private credit of the banking sector over GDP | Beck and Levine (2005) |
| Liquid liabilities of the banking sector over GDP | Beck and Levine (2005) |
| Business cycles | |
| Average of first and second lags of real GDP growth | WDI (2006) |
| Average of first and second lags of real credit growth | IFS (2006) |

Table A1

*(continued)*

| Variable | Source |
|---|---|
| Financial account openness | |
| Lag of absolute gross inflows/GDP | IFS (2006) |
| Lag of sum of absolute gross inflows and absolute Gross outflows/GDP | IFS (2006) |
| Stocks of foreign assets and foreign liabilities | Lane and Milesi-Ferretti (2006) |
| Lag of net foreign assets | Lane and Milesi-Ferretti (2006) |
| Lag of stock of foreign liabilities/GDP | Lane and Milesi-Ferretti (2006) |
| Lag of stock of debt liabilities/stock of liabilities | Lane and Milesi-Ferretti (2006) |
| Lag of stock of FDI/stock of liabilities | Lane and Milesi-Ferretti (2006) |
| Governance | |
| Lag of law and order index | ICRG (2005) |
| Lag of government stability index | ICRG (2005) |
| Others | |
| Ratio of foreign liabilities to money in the financial sector | IFS (2006) |

*Notes.* IFS, International Financial Statistics; GDF, Global Development Finance; WDI, World Development Indicators; ICRG, International Country Risk Guide.

*Proof of Lemma 1.* The external credit constraint (4) is binding in normal times if the marginal utility of consumption remains higher than the expected marginal utility of consumption in the next period, that is, if

$$u'(C_t^n) > (1-\pi)\ u'(C_{t+1}^n) + \pi\ u'(C_{t+1}^s).$$

Using equation (18) this condition can be rewritten as:

$$u'(C_t^n) > \left[1 + \pi\left(\frac{1}{p} - 1\right)\right] u'(C_{t+1}^n),$$

and using the constant relative risk aversion (CRRA) specification (2), as well as the fact that consumption grows at rate $g$ before the sudden stop, we obtain (20).

During a sudden stop episode, the consumption path is deterministic and the external credit constraint (4) is binding if consumption increases over time. For a sudden stop starting at time $t$, this means

$$C_t \leq C_{t+1} \leq \cdots \leq C_{t+\theta} \leq C_{t+\theta+1}.$$

An expression for $C_{t+\tau}$ can be derived, for $\tau = 1, \ldots, \theta + 1$, by using (3) with $Z_t = 0$, (4) as an equality and (5) and (9),

$$\begin{aligned} C_{t+\tau} &= Y_{t+\tau}^s + \frac{\alpha(\tau)}{1+r} Y_{t+\tau+1}^n - \alpha(\tau-1) Y_{t+\tau}^n, \\ &= \left[1 - \gamma(\tau) + \frac{1+g}{1+r}\alpha(\tau) - \alpha(\tau-1)\right] Y_{t+\tau}^n. \end{aligned} \tag{A.1}$$

This formula also applies for $\tau = \theta + 1$ if we define $\gamma(\theta+1) = 0$ and $\alpha(\theta+1) = \alpha(\theta) = \alpha$.

Using (A.1) the inequality $C_{t+\tau} \leq C_{t+\tau+1}$ can be written, for $t = 1, \ldots, \theta$ as:

$$1 - \gamma(\tau) - \frac{r-g}{1+r}\alpha(\tau) + \alpha(\tau) - \alpha(\tau-1) \leq (1+g)\left\{1 - \gamma(\tau+1) + \frac{1+g}{1+r}[\alpha(\tau+1) - \alpha(\tau)] - \frac{r-g}{1+r}\alpha(\tau)\right\}.$$

Because $\alpha(\tau + 1) - \alpha(\tau) \geq 0$ and $\gamma(\tau + 1) \leq \gamma(\tau)$ this inequality is necessarily satisfied if,

$$1 - \gamma(\tau) - \frac{r-g}{1+r}\alpha(\tau) + \alpha(\tau) - \alpha(\tau - 1) \leq (1+g)\left[1 - \gamma(\tau) - \frac{r-g}{1+r}\alpha(\tau)\right],$$

or

$$\alpha(\tau) - \alpha(\tau - 1) \leq g\left[1 - \gamma(\tau) - \frac{r-g}{1+r}\alpha(\tau)\right],$$

which in turn is true if (21) is satisfied (because $\gamma(\tau) \leq \gamma$ and $\alpha(\tau) \leq \alpha$). Note that under the linear specification $\alpha(\tau)=\alpha(\tau/\theta)$, condition (21) implies a lower bound on the duration of a sudden stop episode

$$\theta \geq \frac{1}{g}\frac{(1+r)\lambda}{(1+g)(1-\gamma) - (r-g)\lambda},$$

(where we used (14) to substitute out $\alpha$).

Finally, we show that $C_t \leq C_{t+1}$. As $C_t = C_t^s \leq C_t^n$ it is sufficient to show that $C_t^n \leq C_{t+1}$. Using

$$C_t^n = \left[1 - \frac{r-g}{1+r}\alpha - \frac{\pi}{\pi + p(1-\pi)}\rho\right]Y_t^n,$$

and (A.1) with $\tau = 1$ and $\alpha(0) = 0$, this is necessarily true if

$$1 - \frac{r-g}{1+r}\alpha \leq (1+g)\left[1 - \gamma(1) + \frac{1+g}{1+r}\alpha(1)\right],$$

which is condition (22).