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Technical Appendix to

MARKET DESIGN AND INVESTMENT INCENTIVES

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Appendices

Proof of Propositions 1 and 4. See proof of Proposition 2 in Fabra *et al.* (2006).

The following result, which is adapted from Theorem 3.1 in Amir *et al.* (2010), is used in the proof of Propositions 3 and 5.

THEOREM 1. *If conditions* C1–C3 *hold, then the simultaneous capacity choice game is a game of strategic substitutes, which has at least one pair of asymmetric pure-strategy equilibria in capacity choices and no symmetric one, where*

C1: π_i^{d-} *and* π_i^{d+} *are submodular.*

C2: $\dfrac{\partial \pi_i^{d+}(k,k)}{\partial k_i} > \dfrac{\partial \pi_i^{d-}(k,k)}{\partial k_i}$ *for all* $k \in (0,1)$.

C3: $\dfrac{\partial \pi_i^{d+}(0,0)}{\partial k_i} > 0$ *and* $\dfrac{\partial \pi_i^{d-}(1,1)}{\partial k_i} < 0$.

If C1 and C2 hold, then payoff functions are submodular on $[0,1]^2$. Due to the kink along the diagonal, the submodularity of π_i^{d-} and π_i^{d+} do not suffice to guarantee overall submodularity of π_i^d. Nevertheless, if C2 holds, then payoff functions are submodular, and, consequently, the capacity game induced by the discriminatory auction is a game of strategic substitutes. Finally, the role of C3 is to rule out that either (0,0) or (1,1) belong to the best-reply correspondences.

Proof of Proposition 3. Since any capacity choice above the maximum demand realisation, $k_i > 1$, is strictly dominated by $k_i = 1$, firm *i*'s strategy space can be constrained to the compact set [0,1]. It follows from Topkis's (1979) Characterisation Theorem that π_i^{d-} and π_i^{d+} are submodular if the second-order cross derivatives are negative, with

$$\frac{\partial^2 \pi_i^{d-}}{\partial k_i \partial k_j} = -\frac{P}{k_j}\left[\int_{k_j}^{k_i+k_j} \frac{\theta - 2k_i}{k_j} \mathrm{d}G(\theta) + g(k_i + k_j)k_i\right],$$

$$\frac{\partial^2 \pi_i^{d+}}{\partial k_i \partial k_j} = -Pg(k_i + k_j) < 0.$$

The second-order cross derivative of the small-firm profit function is always negative if $k_j \geq 2k_i$, as then $\int_{k_j}^{k_i+k_j} (\theta - 2k_i)/k_j \, \mathrm{d}G(\theta) > 0$. If $\theta g(\theta)$ is increasing, the result also holds for pairs (k_i, k_j) such that $k_i \leq k_j < 2k_i$. To see this, note that we can write $\partial^2 \pi_i^{d-}/\partial k_i \partial k_j = -(P/k^+)/H(k_i, k_j)$, where

$$
\begin{aligned}
H(k_i,k_j) &= \int_{k_j}^{k_i+k_j} \frac{\theta-2k_i}{k_j}\,\mathrm{d}G(\theta) + g(k_i+k_j)k_i \\
&= \frac{1}{k_j}\int_{k_j}^{k_i+k_j} \{G(k_i+k_j)+k_j g(k_i+k_j) - [G(\theta)+(2k_i-k_j)g(\theta)]\}\,\mathrm{d}\theta \\
&\geq \frac{1}{k_j}\int_{k_j}^{k_i+k_j} \{G(k_i+k_j)+k_j g(k_i+k_j) - [G(\theta)+k_j g(\theta)]\}\,\mathrm{d}\theta.
\end{aligned}
$$

The second equality follows from straightforward computations after integration by parts of $\theta g(\theta)$, and the inequality from the fact that $2k_i - k_j \leq k_j$. A sufficient condition for the integral above to be positive is that $G(\theta) + k_j g(\theta)$ be an increasing function in θ on $(k_j, k_i + k_j)$, which holds trivially if $g(\theta) + \theta g(\theta) \geq 0$, that is, $\theta g(\theta)$ is increasing.

Hence, condition C1 in Theorem 3 holds. Equation (4) shows that condition C2 holds. Last, since $[\partial \pi_i^{d+}(0,0)]/[\partial k_i] = P - c > 0$ and $\partial \pi_i^{d-}(1,1)/\partial k_i = -c < 0$, condition C3 in Theorem 3 also holds. Therefore, the capacity game is a game of strategic substitutes, it has at least one pair of asymmetric pure-strategy equilibria and no symmetric one.

The equilibrium is unique since the slope of the best correspondence function of the small firm is smaller than the one corresponding to the large firm so that they can cross only once, that is,

$$
\left|\frac{\partial^2 \pi_i^{d-}}{\partial k^- \partial k^+}\right| - \left|\frac{\partial^2 \pi_i^{d+}}{\partial k^+ \partial k^-}\right| < 0
$$

holds. To see this, note that

$$
\left|\frac{\partial^2 \pi_i^{d-}}{\partial k^- \partial k^+}\right| - \left|\frac{\partial^2 \pi_i^{d+}}{\partial k^+ \partial k^-}\right| = P\left[\frac{1}{k^+}\int_{k^+}^{K} \frac{(\theta-2k^-)}{k^+}\,\mathrm{d}G(\theta) - g(K)\left(\frac{k^+-k^-}{k^+}\right)\right].
$$

Furthermore,

$$
\frac{1}{k^+}\int_{k^+}^{K} \frac{(\theta-2k^-)}{k^+}\,\mathrm{d}G(\theta) < \frac{1}{k^+} g(K)\left(\frac{k^+-k^-}{k^+}\right)k^- < g(K)\left(\frac{k^+-k^-}{k^+}\right),
$$

where the first inequality follows from the fact that $(\theta - 2k^-)g(\theta)$ is an increasing function for any $\theta \geq 2k^-$(if g is non-increasing then $\theta g(\theta)$ increasing implies that $(\theta - 2k^-)g(\theta)$ is increasing as well, whereas if g is increasing the result follows from $\theta - 2k^- \geq 0$) and the second inequality from the fact that $k^- < k^+$. It hence follows that

$$
\left|\frac{\partial^2 \pi_i^{-}}{\partial k^- \partial k^+}\right| - \left|\frac{\partial^2 \pi_i^{+}}{\partial k^+ \partial k^-}\right| < 0,
$$

as claimed.

Proof of Proposition 5. Following similar steps as those in the proof of Lemma 3 in Fabra *et al.* (2006), it is possible to characterise the unique mixed strategies equilibrium in the uniform-price auction when firms face demand uncertainty at the pricing stage. Let $F_j^u(b) = \Pr(b_j \leq b)$ denote the equilibrium mixed-strategy of firm j, $j = 1,2$. For any price in the support, $b \in [\underline{b}^u, P)$,

$$F_j^u(b) = \begin{cases} \beta_j \ln\left(\dfrac{b}{\underline{b}^u}\right) & \text{for } \lambda_j = 0 \\ \dfrac{\beta_j}{\lambda_j}\left[\left(\dfrac{b}{\underline{b}^u}\right)^{\lambda_j} - 1\right] & \text{for } \lambda_j \neq 0 \end{cases}$$

where

$$\lambda_j = \frac{\int_{k_j}^{1} \min(\theta - k_j, k_i)\,\mathrm{d}G(\theta) - \int_0^{k_i} \theta\,\mathrm{d}G(\theta)}{\int_0^1 \min(\theta, k_i)\,\mathrm{d}G(\theta) - \int_{k_j}^1 \min(\theta - k_j, k_i)\,\mathrm{d}G(\theta)},$$

$$\beta_j = \frac{\int_0^{k_i} \theta \mathrm{d}G(\theta)}{\int_0^1 \min(\theta, k_i)\mathrm{d}G(\theta) - \int_{k_j}^1 \min(\theta - k_j, k_i)\mathrm{d}G(\theta)}.$$

If $k_i \leq k_j$ then $\beta_i \geq \beta_j$, and if $k_1 = k_2$, then $\beta_1 = \beta_2$ and $\lambda_1 = \lambda_2$.

If λ_j, $\lambda_i \neq 0$ and if $\lambda_j \geq \lambda_i$ and $(\beta_j/\lambda_j) \geq (\beta_i/\lambda_i)$, it follows that

$$\lim_{b\uparrow P} F_j^u(b) = \frac{\beta_j}{\lambda_j}\left[\left(\frac{b}{\underline{b}^u}\right)^{\lambda_j} - 1\right] \geq \frac{\beta_i}{\lambda_i}\left[\left(\frac{b}{\underline{b}^u}\right)^{\lambda_j} - 1\right] = \lim_{b\uparrow P} F_i^u(b).$$

Since at most one player can bid P with positive probability, it follows that in this cases we must have $\lim_{b\uparrow P} F_i^u(b) \leq \lim_{b\uparrow P} F_j^u(b) = 1$. Then it is straightforward to verify that $\underline{b}^u$ is given uniquely by

$$\underline{b}^u = P\left(\frac{\beta_j}{\beta_j + \lambda_j}\right)^{1/\lambda_j}. \tag{A1}$$

Equilibrium profits are, for $j = 1, 2$,

$$\pi_j^u = P\left[\Pr(b_i < P)\int_{k_i}^1 \min(\theta - k_i, k_j)\,\mathrm{d}G(\theta) + \Pr(b_i = P)\int_0^1 \min(\theta, k_j)\,\mathrm{d}G(\theta)\right],$$

where $\Pr(b_i < P) = \lim_{b\uparrow P} F_i^u(b)$.

We now analyse the behaviour of the equilibrium as one considers the limit case in which demand uncertainty vanishes out and the price subgame converges to the complete information price subgame. This is accomplished by performing comparative statics as we let the distribution of θ converge to a Delta-Dirac distribution. By fixing the 'spike' or the mass point at $\theta = \theta_C$, we can study the limiting behaviour of the mixed strategy-equilibrium as $[g_\sigma(\theta)] \to \delta_{\theta_C}$, where $0 \leq \theta_C \leq 1$. The following properties of the Delta-Dirac will be used throughout this proof:

(*i*) The delta function obeys the sifting property: $\int_a^b h(y)\delta(y - \theta_C)\mathrm{d}y = h(\theta_C)$ (Bracewell, 1999, pp. 74–5) so that $\int_0^1 y g_\sigma(y)\mathrm{d}y \to \theta_C$.

(*ii*) Whenever the sifting property yields indeterminate values for the parameters in F^u, we will use the fact that the Delta-Dirac function can be expressed as the limit (in the sense of distributions) of the sequence of Gaussians $\delta_\sigma(x) = 1/(\sigma\sqrt{\pi})\mathrm{e}^{-(x-\theta)^2/\sigma^2}$ as $\sigma \to 0$.

Using the above results to compute the values of parameters μ_j, λ_j and β_j as g converges to δ, we next analyse the convergence of the unique mixed-strategy equilibrium (MSE) of the pricing game under the uniform-price auction when θ is almost certainly in (k_2, K), that is, High Demand Region II.

Since $\beta_1 \to \beta_2 \to 0$, $F_j^u(b) \to 0$ for all j unless $\underline{b}^u \to 0$. We first explore if this is indeed the case.

Assume first that $\lim_{b\uparrow P} F_2^u(b) \le \lim_{b\uparrow P} F_1^u(b) = 1$ which, together with $\lambda_1 \to (\theta - k_1)/(k_1 + k_2 - \theta) \ne 0$, imply that $\underline{b}^u$ is uniquely given by A1), with

$$\underline{b}^u = P\left(\frac{\beta_1}{\lambda_1 + \beta_1}\right)^{1/\lambda_1} \to 0, \quad \text{as } \beta_1 \to 0.$$

Since $\underline{b}^u \to 0$, then

$$\lim_{b\uparrow P} F_2^u(b) = \frac{\beta_2}{\lambda_2}\left(\frac{\lambda_1 + \beta_1}{\beta_1}\right)^{\lambda_2/\lambda_1} - \frac{\beta_2}{\lambda_2} \to 0$$

as $\beta_2 \to 0$, $\lambda_2 \to (\theta - k_2)/(k - \theta) > 0$ and

$$\frac{\beta_2}{\lambda_2}\left(\frac{\lambda_1 + \beta_1}{\beta_1}\right)^{\lambda_2/\lambda_1} = \lim_{\sigma \to 0}\left\{\left[\frac{\int_0^x z e^{-[(z+y-\theta)/\sigma]^2} dz + x\int_{x+y}^1 e^{-[(z-\theta)/\sigma]^2} dz}{\int_0^x z e^{-[(z-\theta)/\sigma]^2} dz} - 1\right]^{-1} \times \left[\frac{\int_0^y z e^{-[(z+x-\theta)/\sigma]^2} dz + y\int_{x+y}^1 e^{-[(z-\theta)/\sigma]^2} dz}{\int_0^y z e^{-[(z-\theta)/\sigma]^2} dz}\right]^{(\theta - k_2)/(\theta - k_1)}\right\} = 0.$$

It hence follows that on the interior of the support $F_2^u(b) \to 0$, while

$$F_1^u(b) = \frac{\lambda_1 + \beta_1}{\lambda_1}\left(\frac{b}{P}\right)^{\lambda_1} - \frac{\beta_1}{\lambda_1} \to \left(\frac{b}{P}\right)^{(\theta - k_1)/(k_1 + k_2 - \theta)}.$$

Therefore, $F_1^u(b)$ converges to the mixed-strategy equilibrium under demand uncertainty at which Pr $(b_1 = P) = 0$, while $F_2^u(b)$ converges to the mixed strategy-equilibrium under demand uncertainty at which Pr $(b_2 = P) = 1$.[1]

Assume next that $\lim_{b\uparrow P} F_2^u(b) = 1 \ge \lim_{b\uparrow P} F_1^u(b)$. We now show that this leads to a contradiction. The value of $\underline{b}^u$ is now given uniquely by (A1) with $\beta_j = \beta_2$ and $\lambda_j = \lambda_2$. Since $\beta_2 \to 0$ and $\lambda_2 \to (\theta - k_2)/(k - \theta) > 0$ then

$$\underline{b}^u = P\left(\frac{\beta_2}{\lambda_2 + \beta_2}\right)^{1/\lambda_2} \to 0, \text{ so that } \lim_{b\uparrow P} F_2^u(b) = \frac{\beta_2}{\lambda_2}\left\{\left[\left(\frac{\lambda_2 + \beta_2}{\beta_2}\right)^{1/\lambda_2}\right]^{\lambda_2} - 1\right\} \to \infty,$$

contradicting $\lim_{b\uparrow P} F_2^u(b) = 1$. Since $\lim_{b\uparrow P} F_2^u(b) \le \lim_{b\uparrow P} F_1^u(b) = 1$ holds, equilibrium approaches something with the flavour of the pure-strategy equilibrium, with the large firm bidding at P and the small firm mixing over a range between 0 and P. Profits converge again to $\pi_1 \to P k_1$ and $\pi_2 \to P(\theta - k_1)$.

[1] Mixed strategy equilibria under demand certainty are derived in Fabra *et al.* (2006).

Proof of Proposition 6. We name Firm 1 the firm with the smallest capacity at the end of stage 1, that is, $i = 1$ iff $k_i \leq k_j$. We let $\hat{k}_1$ denote the capacity choice of Firm 1 at the end of stage 2, and $\hat{k}_2$ the capacity choice of Firm 2 at the end of the third stage. Since at stages 2 and 3 firms can only reduce or mothball capacity, it follows that $\hat{k}_i \leq k_i$ $i = 1,2$. Stage 3 determines firms' identities (either small or large) so that, whenever clear from the context, we will write $\hat{k}_2 = k^-$ or $\hat{k}_2 = k^+$.

We solve the game by backward induction, beginning with the optimal choice of Firm 2 at stage 3. If Firm 2 were to become the small firm then $\hat{k}_2 = k^- = \min(k^*, \hat{k}_1 - \epsilon)$, where $k^* = \arg\max_k Pk[1 - G(k)]$ (recall that investment costs are sunk at this stage). If Firm 2 opts for being the large firm, its expected profits are independent of k_2 whenever $\hat{k}_1 + k_2 > 1$, whereas they are increasing in k_2 otherwise. Consequently, Firm 2's constrained maximum as large firm is $\hat{k}_2 = k^+ = k_2$. To set the best reply we must compare profits at $\hat{k}_2 = k^-$ and at $\hat{k}_2 = k^+$. Let $\bar{k}^+(k^-)$ stand for the capacity of the large firm that yields the same revenues as k^-. Formally,

$$\bar{k}^+(k^-) = \left\{ k^+ \in (k^-, 1) : \int_{k^-}^{1} k^- \mathrm{d}G(\theta) = \int_{k^-}^{K} (\theta - k^-) \mathrm{d}G(\theta) + \int_{K}^{1} k^+ \mathrm{d}G(\theta) \right\}.$$

If no k^+ satisfies equality above, we will set $\bar{k}^+(k^-) = 1$.

The best reply of Firm 2 at Stage 3 is hence:

$$R_2(\hat{k}_1, k_2) = \begin{cases} \min(k^*, \hat{k}_1 - \epsilon) & \text{if } k_2 < \bar{k}^+(\hat{k}_1) \\ k_2 & \text{if } k_2 \geq \bar{k}^+(\hat{k}_1). \end{cases}$$

Consider next the best reply by Firm 1 at Stage 2. By backward induction if $k_2 \geq \bar{k}^+(k_1)$, the best Firm 1 can do is to maintain its existing capacity given that its rival will respond by maintaining its own. If $k_2 < \bar{k}^+(k_1)$, Firm 2 will undercut. To maintain the status of small firm, Firm 1 has to make undercutting unprofitable to its rival by setting $\hat{k}_1$ that satisfies $k_2 = \bar{k}^+(\hat{k}_1)$. Reducing to $\hat{k}_1$ is more profitable than keeping k_1 as

$$\pi^-[\hat{k}_1, \bar{k}^+(\hat{k}_1)] = \pi^+(\hat{k}_1, k_2) > \pi^+(k^- = k_1 - \epsilon, k^+ = k_2) > \pi^+(k^- = k_1 - \epsilon, k^+ = k_1),$$

where the equality follows from the definition of $\bar{k}^+(\hat{k}_1)$, the first inequality follows from the fact that π^+ is decreasing in k^- for any k^+, and the second one from the fact that over this region the large firm profits are increasing in its own capacity. Thus Firm 1's best reply is given by

$$R_1(k_1, R_2) = \min[k_1, \hat{k} : k_2 = \bar{k}^+(\hat{k})].$$

It follows from R_1 and R_2 that, at any equilibrium, the firm that produces less in the first stage will maintain the status of small firm, whereas its rival will be the large firm. As such it will produce in the first stage so as to be on its FOC given that π^+ is a strictly concave function. Consequently, an equilibrium is a capacity pair $(\tilde{k}^-, \tilde{k}^+)$ that solves system formed by (5) and (6). To see that it is a Nash equilibrium of the overall game we show next that there is no profitable deviation for the small firm given R_1 and R_2 and $k_2 = \tilde{k}^+$. Since payoffs for firm i will depend on market revenues which are a function of capacity choices at the end of stage 3, $(\hat{k}_i, \hat{k}_j)$, and on capacity costs, which are a function of firms' choices at the first stage, (k_i, k_j), it follows trivially that

$$\pi^-(\tilde{k}^-, \tilde{k}^+) > \pi^-(k^- < \tilde{k}^-, \tilde{k}^+) \quad \text{since } \tilde{k}^- \text{ is closer to } k^*,$$

and

$$\pi^-(\tilde{k}^-, \tilde{k}^+) > \pi^-(k^- > \tilde{k}^-, \tilde{k}^+) = \pi^-(\tilde{k}^-, \tilde{k}^+) + c(k^- - \tilde{k}^-).$$

Similarly, there is no profitable deviation for the large firm given R_1 and R_2, as $\tilde{k}^+$ maximises $\pi^+(\tilde{k}^-, k^+)$ and $\hat{k}_i = k_i$ for both firms.

Last, equilibrium existence is guaranteed: since at $k^{u-} = 0 < k^{u+}$, the left-hand side of (6) is smaller than the right-hand side, while the opposite is true at $k^{u-} = k^{u+}$, it follows that for any given k^{u+}, there exists at least one k^{u-} that solves (6).

Proof of Proposition 8. Under uniformly distributed demand, firms' expected profits conditionally on being large or small are concave in own's capacity. Hence, equilibrium capacity choices (k^{d+}, k^{d-}) can be found by solving the system of FOCs (2) and (3). Writing $z = c/P$, these can be simplified to

$$1 - z = k^{d-}\left[1 - 2\ln\left(\frac{k^{d-}}{k^{d+}}\right) + \frac{3}{2}\frac{k^{d-}}{k^{d+}}\right], \tag{A2}$$

$$1 - z = k^{d+} + k^{d-}. \tag{A3}$$

To characterise equilibria fully, let $k^{d+} = \alpha k^{d-}$, with $\alpha > 1$. Using (A3), we find

$$k^{d-} = \frac{1}{1+\alpha}(1-z).$$

From this result (A2), we find $\alpha^2 - 2\alpha\ln(\alpha) - \frac{3}{2} = 0$, which has a unique solution, $\alpha \simeq 2.34$. Aggregate equilibrium capacity is $K^d = 1 - z$, and equilibrium profits become

$$\pi_i^{d-} + \pi_i^{d+} = \mu P\,(1-z)^2,$$

where

$$\mu = \frac{\alpha^3 + 2\alpha^2 - 1 - 2\alpha\ln\alpha}{2\alpha(1+\alpha)^2} \simeq 0.36.$$

In the uniform-price auction, equilibrium capacity choices (k^{u+}, k^{u-}) solve the system of (5) and (6), which for uniformly distributed demand can be simplified to

$$k^{u+} + k^{u-} = 1 - z,$$

$$k^{u-}(1 - k^{u-}) = \frac{1}{2}k^{u+}(2 - 2k^{u-} - k^{u+}).$$

The solution is

$$k^{u-} = \frac{2}{3}\left(1 - \frac{1}{2}\sqrt{1+3z^2}\right),$$

$$k^{u+} = \frac{1}{3}\left(1 - 3z + \sqrt{1+3z^2}\right).$$

Aggregate equilibrium capacity is $K^u = 1 - z$, and aggregate equilibrium profits become

$$\pi_i^{u-} + \pi_i^{u+} = \frac{2}{9}P\left[1 - \frac{3}{2}z(3-z) + \sqrt{1+3z^2}\right].$$

Part (*i*) follows from comparing equilibrium capacities, as derived above. Let $\Delta k^- = k^{u-} - k^{d-}$ denote the difference in the small firm's capacity across the two auction formats. We have

$$\frac{\mathrm{d}\Delta k^-}{\mathrm{d}z} = \frac{1}{1+\alpha} - z(1+3z^2)^{-\frac{1}{2}},$$

$$\frac{\mathrm{d}^2\Delta k^-}{\mathrm{d}z^2} = -(1+3z^2)^{-\frac{3}{2}} < 0.$$

In other words, Δk^{-} is increasing in z for $z < (\alpha^2 + 2\alpha - 2)^{-\frac{1}{2}} \simeq 0.35$ – where it reaches its maximum value (0.08) – and it is declining thereafter to equal 0 at $z = 1$, implying that $k^{d-} < k^{u-}$ for all $z \in [0,1)$. Since aggregate capacity is the same for both auction formats, it follows that $k^{d+} > k^{u+}$. So, the discriminatory auction results in more capacity asymmetry than the uniform-price auction.

Part (*ii*) follows from comparing equilibrium aggregate profits. Let $\Delta\pi = (\pi_i^{u-} + \pi_i^{u+}) - (\pi_i^{d-} + \pi_i^{d+})$. We then have

$$\Delta\pi = \frac{2}{9}P\left[1 - \frac{3}{2}z(3 - z) + \sqrt{1 + 3z^2}\right] - \mu P(1 - z)^2, \tag{A4}$$

$$\frac{\mathrm{d}\Delta\pi}{\mathrm{d}z} = -\frac{2}{3}P\left[\frac{3}{2} - z - 3\mu(1 - z) - z(1 + 3z^2)^{-\frac{1}{2}}\right],$$

$$\frac{\mathrm{d}^2\Delta\pi}{\mathrm{d}z^2} = -\frac{2}{3}P\left[3\mu - 1 - (1 + 3z^2)^{-\frac{3}{2}}\right].$$

From the observations that $(\mathrm{d}^2\Delta\pi)/(\mathrm{d}z^2)$ is decreasing in z and $(\mathrm{d}^2\Delta\pi)/(\mathrm{d}z^2) \simeq 0.03P$ at $z = 1$, it follows that $(\mathrm{d}^2\Delta\pi)/(\mathrm{d}z^2) > 0$, that is, $\Delta\pi$ is concave in z. Given this result and the fact that $(\mathrm{d}\Delta\pi)/(\mathrm{d}z) = 0$ at $z = 1$, we must have $(\mathrm{d}\Delta\pi)/(\mathrm{d}z) < 0$, that is, $\Delta\pi$ is decreasing in z. Then, since $\Delta\pi = 0$ at $z = 1$, it follows that $\Delta\pi > 0$ for all $z \in [0, 1)$.

Finally, note that consumer payments equal total revenues of firms (the sum of capacity costs and profits). As capacity costs are the same in both auctions, the fact that profits are lower with the discriminatory format implies that consumer payments are lower also. Moreover, since expected consumption is the same with both formats, the average price is lower and consumer surplus is higher in the discriminatory auction.

Proof of Proposition 9. Consumer surplus equals total welfare less aggregate profits, or $CS^a = W^a - (\pi_i^{a-} + \pi_i^{a+})$.

Since aggregate capacities are the same, so is total welfare. Writing $z = c/P$ we have

$$\begin{aligned}W^a &= v\left[\int_0^{k^{a-}+k^{a+}} \theta\,\mathrm{d}\theta + \int_{k^{a-}+k^{a+}}^{1} (k^{a-} + k^{a+})\mathrm{d}\theta\right] - c(k^{a-} + k^{a+}) \\ &= v\left[\int_0^{1-z} \theta\,\mathrm{d}\theta + \int_{1-z}^{1} (1 - z)\mathrm{d}\theta\right] - c(1 - z) = \frac{1}{2}(1 - z)[v(1 + z) - 2c].\end{aligned}$$

Under the discriminatory auction, we find

$$\begin{aligned}\frac{\mathrm{d}CS^d}{\mathrm{d}P} &= z^2\left(\frac{v}{P} - 1\right) - \mu(1 - z^2) \\ \frac{\mathrm{d}^2 CS^d}{\mathrm{d}P^2} &= -\frac{c^2}{P^3}\left(3\frac{v}{P} - 2 + 2\mu\right)\end{aligned}$$

where $\mu \simeq 0.36$. It follows that, given $P \leq v$, $\mathrm{d}^2 CS^d/\mathrm{d}P^2 < 0$. Moreover, $\mathrm{d}CS^d/\mathrm{d}P = (v/c) - 1 > 0$ at $P = c$ and $\mathrm{d}CS^d/\mathrm{d}P = -\mu[1 - (c/v)^2] < 0$ at $P = v$. Consequently, CS^d is maximised for some $P^d \in (c, v)$.

Under the uniform-price auction, we find

$$\frac{\mathrm{d}CS^u}{\mathrm{d}P} = z\left(\frac{v}{P} - \frac{2}{3}\right) - \frac{2}{9}\left(1 + \frac{1}{\sqrt{1+3z^2}}\right),$$

$$\frac{\mathrm{d}^2 CS^u}{\mathrm{d}P^2} = -\frac{c^2}{3P^3}\left[9\frac{v}{P} - 4 + 2(1+3z^2)^{-\frac{3}{2}}\right].$$

Given $P \leq v$, $\mathrm{d}^2CS^u/\mathrm{d}P^2 < 0$. Moreover, $\mathrm{d}CS^u/\mathrm{d}P = (v/c) - 1 > 0$ at $P = c$ and $\mathrm{d}CS^u/\mathrm{d}P = \frac{1}{3}(c/v) - \frac{2}{9}\{1 + [1 + 3(c/v)^2]^{-\frac{1}{2}}\} < 0$ at $P = v$. Consequently, CS^u is maximised for some $P^u \in (c,v)$.

Last, since $\Delta CS = CS^d - CS^u = \Delta\pi$, where $\Delta\pi$ is given by (A4), and $\Delta\pi$ is decreasing in c/P, it follows that ΔCS is increasing in P, or $\mathrm{d}CS^d/\mathrm{d}P > \mathrm{d}CS^u/\mathrm{d}P$. From this, we conclude that $P^d > P^u$.

Proof of Proposition 10. The unique mixed-strategy equilibrium of the uniform-price auction with demand uncertainty at the pricing stage has been characterised in Proposition 5. In the discriminatory auction, following similar steps as those in the proof of Lemma 3 in Fabra *et al.* (2006), it is also possible to show that the equilibrium mixed-strategy of firm j, $F_j^d(b) = \Pr(b_j \leq b)$, for any price b in the support $(\underline{b}^d, P]$, is given by

$$F_j^d(b) = \frac{\int_0^1 \min(\theta, k_i)\mathrm{d}G(\theta)}{\int_0^1 \min(\theta, k_i)\mathrm{d}G(\theta) - \int_{k_j}^1 \min(\theta - k_j, k_i)\mathrm{d}G(\theta)}\left(\frac{b - \underline{b}^d}{b}\right).$$

From the observation that denominators in $F_i^d(b)$ and $F_j^d(b)$ are identical, it follows that $k_i \leq k_j$ implies $F_i^d(b) \geq F_j^d(b)$. Equilibrium profits become, for $j = 1,2$,

$$\pi_j^d = P\left[\Pr(b_i < P)\int_{k_i}^1 \min(\theta - k_i, k_j)\mathrm{d}G(\theta) + \Pr(b_i = P)\int_0^1 \min(\theta, k_j)\mathrm{d}G(\theta)\right]$$

where $\Pr(b_i < P) = \lim_{b\uparrow P} F_i^d(b)$.

Proof of Proposition 11. Let k^- and k^+ denote the capacities of the smaller and larger firm, respectively and define F^{a-} and F^{a+} correspondingly, for $a = d, u$. Consider first the discriminatory auction. For uniformly distributed demand, the mixed-strategy distribution functions becomes

$$F_j^d(b) = \left(\frac{2 - k_i}{2k_j}\right)\left(\frac{b - \underline{b}^d}{b}\right).$$

From $F^{d-}(b) \geq F^{d+}(b)$, we have the boundary condition $\lim_{b\uparrow P} F^{d-}(b) = 1$, from which it follows that $\underline{b}^d = P(2-2k^- - k^+)/(2-k^+)$.

Equilibrium profits become

$$\pi_i^{d-} = \frac{1}{2}Pk^-\frac{2-k^-}{2-k^+}(2 - 2k^- - k^+) - ck^-,$$

$$\pi_i^{d+} = \frac{1}{2} Pk^+(2 - 2k^- - k^+) - ck^+.$$

Marginal profits are given by

$$\frac{\partial \pi_i^{d-}}{\partial k^-} = P \frac{(1 - k^-)(2 - 2k^- - k^+) - k^-(2 - k^-)}{2 - k^+} - c,$$

$$\frac{\partial \pi_i^{d+}}{\partial k^+} = P(1 - k^- - k^+) - c.$$

The latter expression implies that at equilibrium aggregate capacity is the same as with short-lived bids. Furthermore,

$$\frac{\partial \pi_i^{d+}(k,k)}{\partial k_i} - \frac{\partial \pi_i^{d-}(k,k)}{\partial k_i} = 2Pk\frac{1-k}{2-k} > 0$$

which rules out symmetric solutions. Now, since best replies are everywhere decreasing functions, if an equilibrium with asymmetric capacities exists, it will be unique. Indeed, such an equilibrium exists. Setting $z = c/P$, closed-form solutions for equilibrium capacities are given by

$$k^{d-} = \frac{1}{2}(2 + z - \sqrt{2 + 4z + 3z^2}), \tag{A5}$$

$$k^{d+} = \frac{1}{2}(-3z + \sqrt{2 + 4z + 3z^2}). \tag{A6}$$

Consider next the uniform-price auction. With uniformly distributed demand, the mixed-strategy distribution functions for $b \in [\underline{b}^u, P)$ become

$$F_j^{u+}(b) = \frac{1}{2}\frac{k^-}{1 - k^- - k^+}\left[\left(\frac{2 - k^+ - 2k^-}{k^+}\right)^{k^-/k^+}\left(\frac{b}{P}\right)^{(1-k^- -k^+)/k^+} - 1\right],$$

$$F_j^{u-}(b) = \frac{1}{2}\frac{k^+}{1 - k^- - k^+}\left[\frac{2 - k^+ - 2k^-}{k^+}\left(\frac{b}{P}\right)^{(1-k^- -k^+)/k^-} - 1\right].$$

Equilibrium profits can also be expressed in terms of k^- and k^+ as follows:

$$\pi_i^{u-} = Pk^-\left\{1 - \frac{1}{2}k^- - \frac{1}{2}\left(\frac{k^-k^+}{1 - k^- - k^+}\right)\left[\left(\frac{2 - 2k^- - k^+}{k^+}\right)^{k^-/k^+} - 1\right]\right\} - ck^-,$$

$$\pi_i^{u+} = Pk^+\left(1 - k^- - \frac{1}{2}k^+\right) - ck^+.$$

Using the profits expression above,

$$\frac{\partial \pi_i^{u-}}{\partial k_i} = P\left\{1 - k^- - \frac{1}{2}\left[\frac{k^-k^+(2 - k^- - 2k^+)}{(1 - k^- - k^+)^2}\right]\left[\left(\frac{2 - 2k^- - k^+}{k^+}\right)^{k^-/k^+} - 1\right]\right.$$
$$\left. - \frac{1}{2}\left[\frac{(k^-)^2}{1 - k^- - k^+}\right]\left(\frac{2 - 2k^- - k^+}{k^+}\right)^{k^-/k^+}\left[\ln\left(\frac{2 - 2k^- - k^+}{k^+}\right) - \frac{2k^-}{2 - 2k^- - k^+}\right]\right\} - c. \tag{A7}$$

$$\frac{\partial \pi_i^{u+}}{\partial k_i} = P(1 - k^- - k^+) - c.$$

The latter expression implies that at equilibrium aggregate capacity is the same as with short-lived bids. Furthermore,

$$\frac{\partial \pi_i^{u+}(k,k)}{\partial k_i} - \frac{\partial \pi_i^{u-}(k,k)}{\partial k_i} = Pk\left[1 + \frac{1}{2}\left(\frac{2-3k}{1-2k}\right)\ln\left(\frac{2-3k}{k}\right)\right] > 0$$

when $0 < 2k < 1$, which rules out existence of symmetric equilibria.

Given the complexity of $\partial \pi_i^{u-}/\partial k_i$, we have not been able to derive closed-form solutions for equilibrium capacities. However, the problem may be solved by numerical methods. From the condition $\partial \pi^{u+}/\partial k_i = 0$, we have $k^+ = 1 - z - k^-$. We define

$$h(k^-) = \frac{1}{P}\frac{\partial \pi^{u-}(k^-, 1-z-k^-)}{\partial k^-}.$$

From (A7) we have that

$$h(k^-) = 1 - k^- - \frac{k^-(1-z-k^-)(2z+k^-)}{2z^2}\left[\left(\frac{1+z-k^-}{1-z-k^-}\right)^{k^-/(1-z-k^-)} - 1\right]$$
$$- \frac{(k^-)^2}{2z}\left(\frac{1+z-k^-}{1-z-k^-}\right)^{k^-/(1-z-k^-)}\left[\ln\left(\frac{1+z-k^-}{1-z-k^-}\right) - \frac{2k^-}{1+z-k^-}\right] - z. \qquad \text{(A8)}$$

A necessary (albeit not sufficient) condition for an equilibrium to exist is that h is downward-sloping and crosses the horizontal axis for some $0 \le k^- \le \frac{1}{2}$. A numerical and graphical examination of h for different values of z demonstrates that this is indeed the case.

Proof of Proposition 12. The proof above shows that equilibrium aggregate capacity is the same under both auction formats, and that it equals $1 - c/P$. Regarding equilibrium capacities, numerical analysis demonstrates that $k^{d-} < k^{u-}$.

We next compare equilibrium profits under the two formats. Total profits in the discriminatory auction are

$$\Pi^d = \pi^{d-} + \pi^{d+} = \frac{1}{2}P(2 - 2k^- - k^+)\frac{(2-k^-)k^- + (2-k^+)k^+}{2-k^+} - c(k^- + k^+).$$

Substituting for k^- and k^+ from (A5) and (A6) above, writing $z = c/P$, we find equilibrium profits in reduced form:

$$\Pi^d = P\left\{\frac{1}{4}\left(z + \sqrt{2+4z+3z^2}\right)\left[\frac{(1+z)\sqrt{2+4z+3z^2} - z(5+4z)}{2 + \frac{1}{2}\left(3z - \sqrt{2+4z+3z^2}\right)}\right] - z(1-z)\right\}. \qquad \text{(A9)}$$

Total profits in the uniform-price auction are

$$\Pi^u = P\left\{(k^- + k^+)\left[1 - \frac{1}{2}(k^- + k^+)\right] - \frac{1}{2}\frac{(k^-)^2 k^+}{1-k^--k^+}\left[\left(\frac{2-2k^--k^+}{k^+}\right)^{k^-/k^+} - 1\right]\right\}$$
$$- c(k^- + k^+).$$

Using the fact that, at equilibrium, $k^- + k^+ = 1 - z$, we may write total profits as a function of k^- alone:

$$\Pi^u(k^-) = P\left\{\frac{1}{2}(1-z^2) - \frac{1}{2}\frac{(k^-)^2(1-z-k^-)}{z}\left[\left(\frac{1+z-k^-}{1-z-k^-}\right)^{k^-/(1-z-k^-)} - 1\right] - z(1-z)\right\}.$$

Numerical analysis demonstrates that profits – and hence prices – are higher with the uniform-price format.

Proof of Proposition 13. Aggregate capacities are the same in all cases, as shown in Propositions 7 and 12.

In the discriminatory format, we can compute closed-form solutions for equilibrium capacity choices and hence equilibrium profits. With short-lived bids, equilibrium capacity of the small firm is given by $[1/(1+\alpha)](1 - z)$, where $z = c/P$ and α is the solution to the equation $\alpha^2 - 2\alpha \ln(\alpha) = \frac{3}{2}$, or $\alpha \approx 2.343164$. With long-lived bids, equilibrium capacity of the small firm is given by $\frac{1}{2}\left(2 + z - \sqrt{2 + 4z + 3z^2}\right)$. Simple algebra shows that the capacity of the small firm with short-lived exceeds that with long-lived bids when $z < 0.07866$, and *vice versa*.

Aggregate profits are given by $0.359987P(1 - z)^2$, when bids are short-lived, whereas in the case of long-lived bids, aggregate profits are given by (A9). Thus, profits – and hence prices – are always lower with short-lived bids. Combining this result with Propositions 8 and 12 gives that aggregate profits – and hence expected prices – are the lowest under the discriminatory auction with short-lived bids.

Last, to support the claim made in the text regarding the effect of bid duration on capacity asymmetries, we compare equilibrium capacity choices in the uniform-price auction. With short-lived bids, equilibrium capacity of the small firm is given by $\frac{2}{3} - \frac{1}{3}\sqrt{1 + 3z^2}$, whereas with long-lived bids the corresponding capacity is given implicitly by the equation $h(k) = 0$, where h is defined in (16). Since $h(\frac{2}{3} - \frac{1}{3}\sqrt{1 + 3z^2}) < 0$ if and only if $z > 0.093$, this – together with the fact that h is decreasing in k – implies that the equilibrium capacity choice of the small firm is smaller with long-lived bids than the corresponding choice with short-lived bids when $z > 0.093$, and *vice versa*.

Proof of Proposition 14. The first-order derivatives of π_i^{u-} and π_i^{u+} are given by

$$\frac{\partial \pi_i^{u-}}{\partial k_i} = P[1 - (2 + \rho_i)k_i] - c,$$
$$\frac{\partial \pi_i^{u+}}{\partial k_i} = P\left(1 - k_i - k_j\right) - c.$$

Since at any equilibrium the FOC of the large firm must be satisfied, it follows that aggregate capacity equals $1 - c/P$, which shows (*ii*).

To show (*i*) we first note that C1–C3 in Theorem 1 hold in our game. First, C1 holds as

$$\frac{\partial^2 \pi^{u+}}{\partial k_i \partial k_j} = -P < 0, \text{ and } \quad \frac{\partial^2 \pi^{u-}}{\partial k_i \partial k_j} = 0.$$

Moreover,

$$\frac{\partial \pi^{u+}(k,k)}{\partial k_i} - \frac{\partial \pi^{u-}(k,k)}{\partial k_i} = \rho_i k > 0,$$

so that C2 holds and best-replies have a downward jump that crosses over the diagonal. Finally,

$$\frac{\partial \pi^{u+}(0,0)}{\partial k_i} = P - c > 0 \text{ and } \frac{\partial \pi^{u-}(1,1)}{\partial k_i} = -P(1 + \rho_i) - c < 0.$$

It hence follows that the best reply of firm i is discontinuous at a unique point $k_j \in (0,1)$.

Setting $\rho_1 = \rho \leq 1/2$ so that $\rho_2 = 1 - \rho$, the two best reply functions in the uniform-price auction become:

$$k_1^u(k_2) = \begin{cases} 1 - \frac{c}{P} - k_2 & \text{if } k_2 < \hat{k}_2 \\ \frac{1}{2+\rho}(1 - \frac{c}{P}) & \text{if } k_2 \geq \hat{k}_2 \end{cases}$$

$$k_2^u(k_1) = \begin{cases} 1 - \frac{c}{P} - k_1 & \text{if } k_1 < \hat{k}_1 \\ \frac{1}{3-\rho}(1 - \frac{c}{P}) & \text{if } k_1 \geq \hat{k}_1, \end{cases}$$

where

$$\hat{k}_2 = \frac{2+\rho-\rho\sqrt{2+\rho}}{(2-\rho)(2+\rho)}\left(1-\frac{c}{P}\right) > \frac{1}{3-\rho}\left(1-\frac{c}{P}\right) \text{ for all } \rho \leq 0.5,$$

and

$$\hat{k}_1 = \frac{3-\rho-(1-\rho)\sqrt{3-\rho}}{(1+\rho)(3-\rho)}\left(1-\frac{c}{P}\right) \leq \frac{1}{2+\rho}\left(1-\frac{c}{P}\right) \text{ for all } \rho \leq 0.27389.$$

If $\rho \in [0, 0.27389]$, the best replies cross only once at

$$k_1^{u-} = \frac{1}{2+\rho}\left(1-\frac{c}{P}\right) < k_2^{u+} = \frac{1+\rho}{2+\rho}\left(1-\frac{c}{P}\right).$$

In the remainder case, $\rho \in (0.27389, 0.5]$, the two best replies cross twice at

$$k_1^{u-} = \frac{1}{2+\rho}\left(1-\frac{c}{P}\right) < k_2^{u+} = \frac{1+\rho}{2+\rho}\left(1-\frac{c}{P}\right)$$

and at

$$k_1^{u+} = \frac{2-\rho}{3-\rho}\left(1-\frac{c}{P}\right) > k_2^{u-} = \frac{1}{3-\rho}\left(1-\frac{c}{P}\right).$$

Details on Equilibrium Characterisation with Price-responsive Demand

Under the uniform-price auction, profits for the small and the large firm are, respectively,

$$\pi_i^{u-} = P\int_{k^-/D(p)}^{1} k^- \mathrm{d}\theta - ck^-,$$

$$\pi_i^{u+} = P\left\{\int_{k^-/D(p)}^{K/D(p)} [\theta D(p) - k^-]\mathrm{d}\theta + \int_{K/D(p)}^{1} k^+ \mathrm{d}\theta\right\} - ck^+.$$

The break-even constraint that determines retail prices can be written as

$$(P-p)K\left[D(p) - \frac{K}{2}\right] = \frac{1}{2}P(k^-)^2$$

from which it follows that

$$\frac{\mathrm{d}p}{\mathrm{d}k^-} = \frac{(P-p)[D(p)-K] - Pk^-}{K[D(p) - \frac{1}{2}K] - (P-p)KD'(p)},$$

$$\frac{\mathrm{d}p}{\mathrm{d}k^+} = \frac{(P-p)[D(p)-K]}{K[D(p) - \frac{1}{2}K] - (P-p)KD'(p)}.$$

In the relevant range, $p < P$ and $D(p) > K$, and so $\mathrm{d}p/\mathrm{d}k^+ > 0$. Also, $\mathrm{d}p/\mathrm{d}k^- > 0$ if k^- is sufficiently small.

In the discriminatory auction, the corresponding profit expressions are

$$\pi_i^{d-} = P\left\{\int_{k^-/D(p)}^{k^+/D(p)} \frac{k^-}{\theta D(p)}[\theta D(p) - k^-]\mathrm{d}\theta + \int_{k^+/D(p)}^{K/D(p)} \frac{k^-}{k^+}[\theta D(p) - k^-]\,\mathrm{d}\theta + \int_{K/D(p)}^{1} k^-\mathrm{d}\theta\right\} - ck^-,$$

$$\pi_i^{d+} = P\left\{\int_{k^-/D(p)}^{K/D(p)} [\theta D(p) - k^-]\,\mathrm{d}\theta + \int_{K/D(p)}^{1} k^+ d\theta\right\} - ck^+.$$

Under the linear demand specification $D(p)=1-\gamma p$, profits may be written as

$$\pi_i^{d-} = (P-c)k^- - \frac{P}{1-p\gamma}\left[\frac{(k^-)^3}{2k^+} + (k^-)^2 + (k^-)^2\ln\frac{k^+}{k^-}\right],$$

$$\pi_i^{d+} = (P-c)k^+ - \frac{P}{1-p\gamma}\frac{(2k^- + k^+)k^+}{2},$$

so that the break-even constraint under the linear demand specification becomes:

$$-2\gamma p^2 + (2P\gamma + 2 - K)p + P\left[K + \frac{(k^-)^2}{k^+} - 2 + \frac{2(k^-)^2}{K}\ln\frac{k^+}{k^-}\right] = 0$$

which has only one admissible solution given by

$$p = \frac{1}{4\gamma}\left(X - \sqrt{X^2 + 8\gamma Y}\right)$$

where

$$X = 2\,P\gamma + 2 - K > 0 \quad \text{and} \quad Y = P\left[K + \frac{(k^-)^2}{k^+} - 2 + 2\frac{(k^-)^2}{K}\ln\frac{k^+}{k^-}\right].$$

We find

$$\frac{\mathrm{d}p}{\mathrm{d}k^-} = -\frac{1}{4\gamma}\left[1 - \frac{X - 4\gamma(\mathrm{d}Y)/(\mathrm{d}k^-)}{\sqrt{X^2 + 8Y\gamma}}\right] = \frac{1}{\sqrt{X^2 + 8\gamma Y}}\left(p - \frac{\mathrm{d}Y}{\mathrm{d}k^-}\right) < 0,$$

$$\frac{\mathrm{d}p}{\mathrm{d}k^+} = -\frac{1}{4\gamma}\left[1 - \frac{X - 4\gamma(\mathrm{d}Y)/(\mathrm{d}k^+)}{\sqrt{X^2 + 8Y\gamma}}\right] = \frac{1}{2\sqrt{X^2 + 8\gamma Y}}\left(p - \frac{\mathrm{d}Y}{\mathrm{d}k^+}\right),$$

where

$$\frac{\mathrm{d}Y}{\mathrm{d}k^-} = P\left[1 + 2\frac{(k^-)^2}{Kk^+} + 2k^-\frac{k^- + 2k^+}{K^2}\ln\frac{k^+}{k^-}\right] > P,$$

$$\frac{\mathrm{d}Y}{\mathrm{d}k^+} = P\left[1 - \left(\frac{k^-}{k^+}\right)^2 + 2\frac{k^+}{K}\left(\frac{k^-}{k^+}\right)^2 - 2\left(\frac{k^-}{K}\right)^2\ln\frac{k^+}{k^-}\right].$$

The sign of $-p + \partial Y/\partial k^+$ depends on the values of k^- and k^+; a sufficient condition for $\mathrm{d}Y/\mathrm{d}k^+ > P$ is $k^+ < ek^-$, in which case $\mathrm{d}p/\mathrm{d}k^+ < 0$ follows.