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Technical Appendix to

AGE-DEPENDENT EMPLOYMENT PROTECTION

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Appendix A. The Existence and Uniqueness of the Equilibrium

Let us rewrite (15) as follows:

$$\frac{c}{\beta(1-\gamma)} = q(\theta)\sum_{i=1}^{T-2}\frac{u_i}{u}\int_{R_{i+1}^0}^{1}[1-G(x)]\mathrm{d}x \equiv f(\theta).$$

First, let us note that $f(\theta)$ is a continuous function, as it is the sum and the product of continuous functions. Secondly, let us notice that $\lim_{\theta\to\infty}q(\theta)=0$. If $f(0)>[c/\beta(1-\gamma)]$, then there is at least one equilibrium. As $R_i^0<b, \forall i$, a sufficient condition for the existence of equilibrium is: $\int_b^1[1-G(x)]\mathrm{d}x>[c/\beta(1-\gamma)]$. Considering an uniform distribution for $G(x)$, it is possible to derive an explicit condition ensuring the existence of the equilibrium characterised in Proposition 1:

$$\frac{1}{2}(1-b)^2\beta(1-\gamma)>c.$$

On the other hand, it is not possible to show an explicit condition under which this equilibrium would be unique. Equation (15) can be rewritten as follows: $\Psi(\theta)=(1-\gamma)\beta\Gamma(\theta)$ with $\Psi(\theta)\equiv[c/q(\theta)]$ and $\Gamma(\theta)\equiv\sum_{i=1}^{T-2}(u_i/u)\int_{R_{i+1}^0}^1[1-G(x)]\mathrm{d}x$. It is straightforward that $\Psi'>0$. The equilibrium is then unique if the function Γ is continuously decreasing:

$$\Gamma'(\theta)=\underbrace{\left.\frac{\partial\Gamma(\theta)}{\partial\theta}\right|_{I(R_{i+1}^0)=Cst}}_{A(?)}+\underbrace{\left.\frac{\partial\Gamma(\theta)}{\partial\theta}\right|_{u_i=Cst}}_{B\leq 0},$$

with $I(R_{i+1}^0)=\int_{R_{i+1}^0}^1[1-G(x)]\mathrm{d}x$.

First, let us note that in MP, we have $A=0$ so that the condition $\Gamma'(\theta)<0$ is always satisfied. Indeed, this first term of the derivative of Γ represents the impact of a variation in θ on the unemployment rates by age: because in MP the agents are homogeneous, this term, related to the non-segmented search, does not exist. In our framework, $A\neq 0$, and it is no longer possible to show that the equilibrium is unique for an arbitrary value of T.

Appendix B. The Complete Model with Persistence

We present the model in the general case where idiosyncratic shocks are persistent. At each age, we then assume that a new productivity ϵ is drawn in the distribution $G(\epsilon)$ with a probability $\lambda\leq 1$. To derive the results without any persistence, $\lambda=1$ must be considered in all equations.

B.1. *The Value Functions of the Agents*

The value functions for an occupied firm, $J_i(\epsilon)$, a worker, $W_i(\epsilon)$, a new firm, $J_i^0(\epsilon)$, a new employed worker, $W_i^0(\epsilon)$ and an unemployed worker are respectively given by:

$$J_i(\epsilon) = \epsilon - w_i(\epsilon) + \beta\left\{\lambda \int_{R_{i+1}}^1 J_{i+1}(x)\mathrm{d}G(x) - \lambda G(R_{i+1})F_{i+1} + (1-\lambda)\max[J_{i+1}(\epsilon); -F_{i+1}]\right\},$$

$$W_i(\epsilon) = w_i(\epsilon) - t_i + \beta\left\{\lambda \int_{R_{i+1}}^1 W_{i+1}(x)\mathrm{d}G(x) + (1-\lambda)\max[W_{i+1}(\epsilon); U_{i+1}] + \lambda G(R_{i+1})U_{i+1}\right\},$$

$$J_i^0(\epsilon) = \epsilon - w_i^0(\epsilon) + \beta\left\{\lambda \int_{R_{i+1}}^1 J_{i+1}(x)\mathrm{d}G(x) - \lambda G(R_{i+1})F_{i+1} + (1-\lambda)\max[J_{i+1}(\epsilon); -F_{i+1}]\right\},$$

$$W_i^0(\epsilon) = w_i^0(\epsilon) - t_i + \beta\left\{\lambda \int_{R_{i+1}}^1 W_{i+1}(x)\mathrm{d}G(x) + (1-\lambda)\max[W_{i+1}(\epsilon); U_{i+1}] + \lambda G(R_{i+1})U_{i+1}\right\},$$

$$U_i = b - t_i + \beta\left\{p(\theta)\int_{R_{i+1}^0}^1 W_{i+1}^0(x)\mathrm{d}G(x) + p(\theta)G(R_{i+1}^0)U_{i+1} + [1-p(\theta)]U_{i+1}\right\}.$$

The productivity thresholds are defined by:

$$J_i(R_i) = -F_i \quad \text{and} \quad J_i^0(R_i^0) = H_i.$$

B.2. *Wage Equations under a Two-tier Structure*

The sharing rules can be written as:

$$-\gamma H_i - (1-\gamma)\mathcal{U}_i = \gamma\left[J_i^0(\epsilon) + \mathcal{W}_i^0(\epsilon)\right] - \mathcal{W}_i^0(\epsilon), \tag{B.1}$$

$$-\gamma F_i - (1-\gamma)\mathcal{U}_i = \gamma[J_i(\epsilon) + \mathcal{W}_i(\epsilon)] - \mathcal{W}_i(\epsilon). \tag{B.2}$$

From the value functions, it turns out that:

$$\begin{aligned} J_i(\epsilon) + W_i(\epsilon) = \epsilon - t_i + \beta\Bigg(&\lambda \int_{R_{i+1}}^1 [J_{i+1}(x) + W_{i+1}(x)]\mathrm{d}G(x) \\ &+(1-\lambda)\{\max[J_{i+1}(\epsilon); -F_{i+1}] + \max[W_{i+1}(x); U_{i+1}]\} \\ &+\lambda G(R_{i+1})U_{i+1} - \lambda G(R_{i+1})F_{i+1}\Bigg). \end{aligned}$$

Using the sharing rules, we deduce that:

$$\begin{aligned} \gamma[J_i(\epsilon) + W_i(\epsilon)] - W_i(\epsilon) = {}& \gamma\epsilon - w_i(\epsilon) + (1-\gamma)t_i - \lambda\gamma G(R_{i+1})\beta F_{i+1} \\ &- (1-\gamma)\lambda\beta U_{i+1} - \gamma\lambda[1 - G(R_{i+1})]\beta F_{i+1} \\ &+ \gamma\beta(1-\lambda)\max[J_{i+1}(\epsilon); -F_{i+1}] \\ &- (1-\gamma)\beta(1-\lambda)\max[W_{i+1}(\epsilon); U_{i+1}]. \end{aligned}$$

We then obtain the following wage equations:

$$\begin{aligned}w_i(\epsilon) = \gamma\epsilon + (1-\gamma)t_i + (1-\gamma)[U_i - \beta U_{i+1}] + \gamma(F_i - \beta F_{i+1})\\ + \gamma\beta(1-\lambda)\max[J_{i+1}(\epsilon) + F_{i+1}; 0] - (1-\gamma)\beta(1-\lambda)\max[W_{i+1}(\epsilon) - U_{i+1}; 0].\end{aligned}$$

Because we also have $\gamma[J_{i+1}(\epsilon)+F_{i+1}] = (1-\gamma)[W_{i+1}(\epsilon)-U_{i+1}]$, and, as $\max[J_{i+1}(\epsilon)+F_{i+1};0] = J_{i+1}(\epsilon)+F_{i+1}$, then $\max[W_{i+1}(\epsilon)-U_{i+1};0] = W_{i+1}(\epsilon)-U_{i+1}$, we finally obtain:

$$\begin{aligned}w_i(\epsilon) &= \gamma(\epsilon + F_i - \beta F_{i+1}) + (1-\gamma)(U_i - \beta U_{i+1})\\ &= \gamma(\epsilon + F_i - \beta F_{i+1} + c\theta\tau_i) + (1-\gamma)b,\end{aligned}$$

given that

$$\begin{aligned}U_i &= b - t_i + \beta\left\{p(\theta)\int_{R^0_{i+1}}^1 \left[W^0_{i+1}(x) - U_{i+1}\right]\mathrm{d}G(x) + U_{i+1}\right\}\\ &= b - t_i + \frac{\gamma}{1-\gamma}c\theta\underbrace{\frac{\int_{R^0_{i+1}}^1 J^0_{i+1}(x)\mathrm{d}G(x)}{\sum_{i=1}^{T-2}(u_i/u)\int_{R^0_{i+1}}^1 J^0_{i+1}(x)\mathrm{d}G(x)}}_{\tau_i} + \beta U_{i+1}.\end{aligned}$$

We can also show, using the same computational method, that:

$$w^0_i(\epsilon) = \gamma(\epsilon + H_i - \beta F_{i+1} + c\theta\tau_i) + (1-\gamma)b.$$

B.3. *The Firm's Value*

Given the solution for the wage, and the free entry condition ($V = 0$), the firm values are, $\forall i \in [1, T-1]$:

$$\begin{aligned}J_i(\epsilon) = \underbrace{(1-\gamma)(\epsilon - b) - \gamma(F_i - \beta F_{i+1}) - \gamma p(\theta)\beta\int_{R^0_{i+1}}^1 [J^0_{i+1}(x) + H_{i+1}]\mathrm{d}G(x)}_{\epsilon - w_i(\epsilon)}\\ + \beta\left\{-\lambda G(R_{i+1})F_{i+1} + \lambda\int_{R_{i+1}}^1 J_{i+1}(x)\mathrm{d}G(x) + (1-\lambda)\max[J_{i+1}(\epsilon); -F_{i+1}]\right\},\end{aligned} \tag{B.3}$$

$$\begin{aligned}J^0_i(\epsilon) = \underbrace{(1-\gamma)(\epsilon - b) - \gamma(H_i - \beta F_{i+1}) - \gamma p(\theta)\beta\int_{R^0_{i+1}}^1 [J^0_{i+1}(x) + H_{i+1}]\mathrm{d}G(x)}_{\epsilon - w^0_i(\epsilon)}\\ + \beta\left\{-\lambda G(R_{i+1})F_{i+1} + \lambda\int_{R_{i+1}}^1 J_{i+1}(x)\mathrm{d}G(x) + (1-\lambda)\max[J_{i+1}(\epsilon); -F_{i+1}]\right\}.\end{aligned} \tag{B.4}$$

We deduce that:

$$J^0_i(\epsilon) = J_i(\epsilon) + \gamma(F_i - H_i).$$

From (B.3) and (B.4), the reservation productivity, defined by $J_i(R_i) = -F_i$, is given by:

$$
\begin{aligned}
-F_i = (1-\gamma)(R_i - b) - \gamma(F_i - \beta F_{i+1}) - \gamma p(\theta)\beta \int_{R^0_{i+1}}^{1} [J^0_{i+1}(x) + H_{i+1}]\mathrm{d}G(x) \\
+ \beta\left\{-\lambda G(R_{i+1})F_{i+1} + \lambda \int_{R_{i+1}}^{1} J_{i+1}(x)\mathrm{d}G(x) + (1-\lambda)\max[J_{i+1}(R_i); -F_{i+1}]\right\}, \qquad \text{(B.5)}
\end{aligned}
$$

and the reservation productivity for a new job, defined by $J^0_i(R^0_i) = -H_i$, is given by:

$$
\begin{aligned}
-H_i =(1-\gamma)(R^0_i - b) - \gamma(H_i - \beta F_{i+1}) - \gamma p(\theta)\beta \int_{R^0_{i+1}}^{1} [J^0_{i+1}(x) + H_{i+1}]\mathrm{d}G(x) \\
+ \beta\left\{-\lambda G(R_{i+1})F_{i+1} + \lambda \int_{R_{i+1}}^{1} J_{i+1}(x)\mathrm{d}G(x) + (1-\lambda)\max[J_{i+1}(R_i); -F_{i+1}]\right\}. \qquad \text{(B.6)}
\end{aligned}
$$

From (B.5) and (B.6), we easily deduce that:

$$
R^0_i = R_i + F_i - H_i.
$$

Using these first results, it is possible to express the value function as a function of the productivity reservation. Taking the difference between (B.3) and (B.5) gives

$$
J_i(\epsilon) =(1-\gamma)(\epsilon - R_i) - F_i + \beta(1-\lambda)\{\max[J_{i+1}(\epsilon); -F_{i+1}] - \max[J_{i+1}(R_i); -F_{i+1}]\}.
$$

The solution to this equation depends on the age-pattern of the productivity thresholds, except for $\lambda = 1$. The strategy in the general case $\lambda \le 1$ is to postulate a monotonic age-pattern and then to verify that the solution for the reservation productivity is consistent with the initial guess.

(*i*) Assuming $R_i > R_{i+1}$, then $J_{i+1}(R_i) > -F_{i+1}$ and $J_{i+1}(\epsilon) > -F_{i+1}$. We deduce that:

$$
J_i(\epsilon) = (1-\gamma)(\epsilon - R_i) - F_i + \beta(1-\lambda)[J_{i+1}(\epsilon) - J_{i+1}(R_i)],
$$

$$
\Leftrightarrow J_i(\epsilon) = -F_i + (1-\gamma)\left[\sum_{j=0}^{T-i-1} \beta^j (1-\lambda)^j\right](\epsilon - R_i).
$$

(*ii*) Assuming $R_i < R_{i+1}$, then $J_{i+1}(R_i) < -F_{i+1}$. This implies that:

$$
J_i(\epsilon) = (1-\gamma)(\epsilon - R_i) - F_i + \beta(1-\lambda)F_{i+1} + \beta(1-\lambda)\max[J_{i+1}(\epsilon); -F_{i+1}].
$$

Because the existing productivity level ϵ can be in the interval $]R_i, R_{i+1}[$, we have two subcases:

(*a*) if $\epsilon \le R_{i+1}$, then $J_{i+1}(\epsilon) \le -F_{i+1}$, and $\max[J_{i+1}(\epsilon); -F_{i+1}] = -F_{i+1}$, implying that:

$$
J_i(\epsilon) = (1-\gamma)(\epsilon - R_i) - F_i,
$$

(*b*) if $\epsilon > R_{i+1}$, then $J_{i+1}(\epsilon) > -F_{i+1}$, implying that:

$$
J_i(\epsilon) = (1-\gamma)(\epsilon - R_i) - F_i + \beta(1-\lambda)F_{i+1} + \beta(1-\lambda)J_{i+1}(\epsilon).
$$

By backward induction, we obtain:

$$
J_i(\epsilon) = -F_i + (1-\gamma)\left\{\sum_{j=0}^{T-i-1} \beta^j (1-\lambda)^j \max[\epsilon - R_{i+j}; 0]\right\}.
$$

The value function can be rewritten as follows:

$$J_i(\epsilon) = -F_i + \mathcal{J}(\epsilon) \quad \text{with } \mathcal{J}(\epsilon) = \begin{cases} \Gamma(\epsilon, \{R_j\}_{j=i+1}^{T-1}) & \text{if} R_i < R_{i+1} \\ \Psi(\epsilon, R_i) & \text{if} R_i > R_{i+1} \end{cases}. \tag{B.7}$$

Let us note that we have simply $J_i(\epsilon) = -F_i + (1-\gamma)(\epsilon - R_i)$ if $\lambda = 1$ whatever the shape of the reservation productivity sequence ($R_i \gtreqless R_{i+1}$).

B.4. *The Reservation Productivity*

Using the general expression ((B.7)) of the value function for an occupied job, the reservation productivity is the solution to:

$$\begin{aligned} -F_i = {} & (1-\gamma)(R_i - b) - \gamma(F_i - \beta F_{i+1}) \\ & - \gamma p(\theta)\beta \int_{R_{i+1}+F_{i+1}-H_{i+1}}^{1} [-F_{i+1} + \mathcal{J}(\epsilon) + \gamma(F_{i+1} - H_{i+1}) + H_{i+1}]\mathrm{d}G(x) \\ & + \beta\Big\{ -\lambda G(R_{i+1})F_{i+1} + \lambda \int_{R_{i+1}}^{1} [-F_{i+1} + \mathcal{J}(\epsilon)]\mathrm{d}G(x). \\ & + (1-\lambda)\max[J_{i+1}(R_i); -F_{i+1}]\Big\}. \end{aligned}$$

Hereafter, we assume that $H_i = F_i$.

- If $R_i < R_{i+1}$, we have $\max[J_{i+1}(R_i);-F_{i+1}] = -F_{i+1}$ and then:

$$R_i = b - F_i + \beta F_{i+1} - [\lambda - \gamma p(\theta)]\beta \sum_{j=0}^{T-i-1} \beta^j (1-\lambda)^j I(R_{i+j}), \tag{B.8}$$

  where $I(R_{i+j}) = \int_{R_{i+j}}^{1} (x - R_{i+j})\mathrm{d}G(x) = \int_{R_{i+j}}^{1} [1 - G(x)]\mathrm{d}(x) > 0$ with $I'(R_{i+j}) < 0$.

- If $R_i > R_{i+1}$, $\max[J_{i+1}(R_i);-F_{i+1}] = J_{i+1}(R_i)$. The solution of the value function leads to:

$$J_{i+1}(R_i) = -F_i + (1-\gamma)\left[\sum_{j=0}^{T-i-2} \beta^j (1-\lambda)^j\right](R_i - R_{i+1}).$$

We then deduce that

$$\begin{aligned} R_i = {} & b - F_i + \beta F_{i+1} - [\lambda - \gamma p(\theta)]\beta P(i+1) I(R_{i+1}) \\ & + (1-\lambda)\beta P(i+1)(R_i - R_{i+1}), \end{aligned} \tag{B.9}$$

where $P(i+1) = \sum_{j=0}^{T-i-2} \beta^j (1-\lambda)^j$.

B.5. *The Dynamic of the Reservation Productivity: Proofs of Propositions* 12, 13, *and* 14

We determine the restrictions under which the reservation productivity increases or decreases with the worker's age. We assume that $H_i = F_i = F$, $\forall i$, implying that $R_i = R_i^0$.

B.5.1 *Proofs of Propositions* 12 *and* 13*: the case without employment protection* $F_i = H_i = 0$

- When the conjecture is $R_i < R_{i+1}$, the sequence of the reservation productivity is given by (B.8). The restriction $R_{T-2} < R_{T-1}$ is satisfied if $\lambda > \gamma p(\theta)$. We deduce that if $R_{i+1} < R_{i+2}$, then $I(R_{i+1}) > I(R_{i+2})$ because $I'(x) < 0$. In this case,

$$R_{i+1} - R_i = -[\lambda - \gamma p(\theta)]\beta \underbrace{[I(R_{i+2}) - I(R_{i+1})]}_{-} + \beta(1-\lambda)\underbrace{(R_{i+2} - R_{i+1})}_{+}.$$

The condition $\lambda > \gamma p(\theta)$ is sufficient to ensure that $R_{i+1} > R_i$.

- When the conjecture is $R_i > R_{i+1}$, the sequence of the reservation productivity is given by (B.9). Then $R_{T-2} > R_{T-1}$ if $\lambda < \gamma p(\theta)$. Using backward iterations, we obtain:

$$[1+\beta(1-\lambda)P(i+1)](R_i - R_{i+1}) = \underbrace{[\gamma p(\theta) - \lambda]\beta[P(i+1)I(R_{i+1}) - P(i+2)I(R_{i+2})]}_{SS_{i,i+1}} + \underbrace{\beta(1-\lambda)P(i+2)(R_{i+1} - R_{i+2})}_{LH_{i,i+1}}.$$

Unambiguously, we have $LH_{i,i+1} > 0$ because from the previous iteration we have $R_{i+1} - R_{i+2} > 0$. $I(R_{i+1}) < I(R_{i+2})$: the strictness of the selection process is more important for the younger workers. On the other hand, $P(i+1) > P(i+2)$ due to the horizon effect. As these two effects go in opposite directions, the sign of $SS_{i,i+1}$ is indeterminate.

As the condition $\lambda < \gamma p(\theta)$ must hold at the first iteration, we deduce that $SS_{i,i+1} > 0$ is a sufficient condition to ensure that $R_i > R_{i+1}$.

B.5.2 *Proof of the Proposition* 14*: the case with constant employment protection* $F = H, \forall i$

When the conjecture is $R_i < R_{i+1}$, using (B.8), $R_{T-2} < R_{T-1}$ if $F < [\lambda - \gamma p(\theta)] \times \int_{b-F}[1-G(x)]\mathrm{d}x$. We then define $\hat{F}(\lambda) = [\lambda - \gamma p(\theta)]\int_{b-\hat{F}(\lambda)}[1-G(x)]\mathrm{d}x$, implying that $\mathrm{d}\hat{F}/\mathrm{d}\lambda > 0$.

Appendix C. The Efficient Allocation

The problem of the planner is to determine the optimal allocation of any worker between the production and the search sectors ($R_i^\star$) and the optimal investment in the search sector ($\theta^\star$). For any $\lambda \leq 1$, the per-unemployed worker value in the search sector and the per-employed worker value in the goods sector are respectively given by:

$$Y_i^s = b - c\theta^\star + \beta\left\{p(\theta^\star)\int_0^1 \max[Y_{i+1}(x); Y_{i+1}^s]\mathrm{d}G(x) + [1-p(\theta^\star)]Y_{i+1}^s\right\}, \tag{C.1}$$

$$Y_i(\epsilon) = \epsilon + \beta\left\{\lambda\int_0^1 \max[Y_{i+1}(x); Y_{i+1}^s]\mathrm{d}G(x) + (1-\lambda)\max[Y_{i+1}(\epsilon); Y_{i+1}^s]\right\}, \tag{C.2}$$

where $c\theta^\star \equiv c(v^\star/u^\star)$ represents the total cost of vacancies ($cv^\star$) per unemployed worker ($u^\star$). The planner's decisions $R_i^\star$, $\forall i$ and $\theta^\star$ are solutions to:

$$\begin{cases} Y_i(R_i^\star) = Y_i^s \\ \theta^\star = \mathrm{Sup}(\sum_i u_i^\star Y_i^u). \end{cases}$$

C.1. *The Efficient Allocation in an Economy Without Persistence: Proof of Proposition* 6

The optimal choice for θ is such that:

$$c\sum_i u_i^\star = cu^\star = p'(\theta^\star)\beta\sum_i u_i^\star\int_{R_{i+1}^\star} S_{i+1}(x)\mathrm{d}G(x). \tag{C.3}$$

Given that $\theta q(\theta) = p(\theta)$ and

$$p'(\theta) = q(\theta)\left[1 + \theta\frac{q'(\theta)}{q(\theta)}\right] = q(\theta)(1-\eta),$$

(C.3) can be rewritten as follows:

$$\frac{c}{q(\theta^\star)} = (1-\eta)\beta\sum_i \frac{u_i^\star}{u^\star}\int_{R_{i+1}^\star}^1 S_{i+1}(x)\mathrm{d}G(x).$$

The optimal reservation productivity at age i can be deduced from $Y_i(R_i^\star) = Y_i^u \Leftrightarrow S_i(R_i) = 0$:

$$0 = R_i^\star - b + c\theta^\star + [1 - p(\theta^\star)]\beta\int_{R_{i+1}^\star}^1 S_{i+1}(x)\mathrm{d}G(x).$$

Since $S_i(\epsilon) = \epsilon - R_i^\star$, we have $\int_{R_{i+1}^\star}^1 S_{i+1}(x)\mathrm{d}G(x) = \int_{R_{i+1}^\star}^1(\epsilon - R_{i+1}^\star)\mathrm{d}G(x) = \int_{R_{i+1}^\star}^1[1-G(x)]\mathrm{d}x$. We then deduce (23) and (24).

C.2. *The Efficient Allocation in an Economy with Persistence: Proof of Proposition* 15

The optimal choice for $\theta^\star$ does not depend directly on the persistence: the first order condition (C.3) holds for any $\lambda \leq 1$. On the other hand, the persistence changes the value of an occupied job. When $\lambda < 1$, the planner's surplus is:

$$S_i(\epsilon) = \epsilon - R_i^\star + (1-\lambda)\beta\{\max[S_{i+1}(\epsilon);0] - \max[S_{i+1}(R_i^\star);0]\}.$$

- If $R_i^\star > R_{i+1}^\star$, then we have $\epsilon > R_i^\star > R_{i+1}^\star$. The surplus is:

$$S_i(\epsilon) = \left[\sum_{j=0}^{T-i-1}\beta^j(1-\lambda)^j\right](\epsilon - R_i^\star) \equiv P(i)(\epsilon - R_i^\star)$$

- If $R_i^\star < R_{i+1}^\star$, then the surplus is

$$S_i(\epsilon) = \sum_{j=0}^{T-i-1}(1-\lambda)^j\beta^j\max(\epsilon - R_{i+j}^\star;0).$$

The reservation productivity R_i of the efficient allocation then differs according to the age-profile that is assumed:

- If $R_i^\star < R_{i+1}^\star$, the reservation productivity is defined by:

$$R_i^\star = b - c\theta^\star - [\lambda - p(\theta^\star)]\beta\int_{R_{i+1}^\star}^1\left[\sum_{j=0}^{T-i-2}(1-\lambda)^j\beta^j\max(x - R_{i+j+1}^\star;0)\right]\mathrm{d}G(x). \tag{C.4}$$

- If $R_i^\star > R_{i+1}^\star$, the reservation productivity is defined by:

$$\begin{aligned} R_i^\star = b - c\theta^\star - [\lambda - p(\theta^\star)]\beta\int_{R_{i+1}^\star}^1 P(i+1)(x - R_{i+1}^\star)\mathrm{d}G(x) \\ - (1-\lambda)\beta P(i+1)(R_i^\star - R_{i+1}^\star). \end{aligned} \tag{C.5}$$

In order to have an interior solution in each case, we assume that $b > c\theta^\star$.

- If $R_i^\star < R_{i+1}^\star$, we have at the first step of the backward iteration:

$$R_{T-2}^\star - R_{T-1}^\star = -[\lambda - p(\theta^\star)]\beta\int_{R_{T-1}^\star}^1\max(x - R_{T-1}^\star;0)\mathrm{d}G(x).$$

The condition $\lambda > p(\theta^\star)$ is sufficient to ensure that $R^\star_{T-2} < R^\star_{T-1}$. In the following iterations, we have:

$$R^\star_{i+1} - R^\star_i = -[\lambda - p(\theta^\star)]\beta \underbrace{\left[I(R^\star_{i+2}) - I(R^\star_{i+1})\right]}_{-} + \beta(1-\lambda)\underbrace{(R^\star_{i+2} - R^\star_{i+1})}_{+}.$$

The restriction $\lambda > p(\theta^\star)$ is sufficient to ensure that the age-increasing dynamics is internally consistent.

- If $R^\star_i > R^\star_{i+1}$, we have at the first step of the backward iteration:

$$(R^\star_{T-2} - R^\star_{T-1})[1 + (1-\lambda)\beta] = -[\lambda - p(\theta^\star)]\beta \int_{R^\star_{T-1}}^{1} \max(x - R^\star_{T-1}; 0)\mathrm{d}G(x).$$

We deduce that the restriction $\lambda < p(\theta^\star)$ is sufficient to ensure that $R^\star_{T-2} < R^\star_{T-1}$. In the following backward iterations, we obtain:

$$\begin{aligned}[1 + \beta(1-\lambda)P(i+1)](R^\star_i - R^\star_{i+1}) = &\underbrace{[p(\theta^\star) - \lambda]\beta\left[P(i+1)I(R^\star_{i+1}) - P(i+2)I(R^\star_{i+2})\right]}_{SS^\star_{i,i+1}} \\ &+ \underbrace{\beta(1-\lambda)P(i+2)(R^\star_{i+1} - R_{i+2})^\star}_{LH^\star_{i,i+1}(+)},\end{aligned}$$

$\lambda < p(\theta^\star)$ is no longer a sufficient condition to ensure that the $R^\star_i$ are monotonously decreasing. Adding the condition $SS^\star_{i,i+1} > 0, \forall i$ is sufficient to ensure that $R^\star_i > R^\star_{i+1}$.

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Comparing on the one hand (C.4) with (B.9) and on the other hand (C.5) with (B.8), it is straightforward to show that $F_i - \beta F_{i+1} = c\theta^\star(\tau^\star_i - 1), \forall\lambda \le 1$ with

$$\tau^\star_i \equiv \frac{\displaystyle\int_{R^\star_{i+1}}^{1} S_{i+1}(x)\mathrm{d}G(x)}{\sum_i (u^\star_i/u^\star)\displaystyle\int_{R^\star_{i+1}}^{1} S_{i+1}(x)\mathrm{d}G(x)}.$$

- If $R^*_i < R^*_{i+1}$, as

$$\left(R^*_{i+1} - R^*_i\right) = [\lambda - p(\theta)]\beta\left[\int_{R^*_{i+1}}^{1} S_{i+1}(x)\mathrm{d}G(x) - \int_{R^*_{i+2}}^{1} S_{i+2}(x)\mathrm{d}G(x)\right],$$

then $\tau^*_i > \tau^*_{i+1}$.

- If $R^*_i > R^*_{i+1}$, the sign of

$$\int_{R^*_{i+1}}^{1} S_{i+1}(x)\mathrm{d}G(x) - \int_{R^*_{i+2}}^{1} S_{i+2}(x)\mathrm{d}G(x) = [P(i+1)I(R^*_{i+1}) - P(i+2)I(R^*_{i+2})],$$

is indeterminate. Proposition 15 shows that a sufficient condition for $R_i > R_{i+1}$ is that the sign of this expression is positive, which implies that $\tau^*_i > \tau^*_{i+1}, \forall i$.