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Technical Appendix to

THE QUALITY OF POLITICAL INSTITUTIONS AND THE CURSE OF NATURAL RESOURCES

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Appendix

A. *Proofs*

Proposition 2. *Human capital induced by the unconstrained solution to the government problem is*

$$e_G = \left[\frac{\kappa(\alpha+\beta)}{\lambda}\right]^{\frac{1}{1-\alpha-\beta}}. \tag{A.1}$$

Therefore, if the government wins the election and revolution is no threat, human capital does not depend on natural resources.

Proof. If the no revolution constraint does not bind (true for sufficiently low values of $\exp(-c/q)$), we have:

$$\begin{aligned}&\max_{e_G,w_G}(W/n - w_G - \lambda e_G + \alpha\kappa e_G^{\alpha+\beta})\\ &\times \min\left(\max\left\{0, \frac{1}{2} + \frac{\ln[w_G + \kappa(1-\alpha)e_G^{\alpha+\beta}] - U_F}{2A}\right\}, 1\right),\end{aligned}$$

and the first order conditions are:

$$0 = -\left\{\frac{1}{2} + \frac{\ln[w_G + \kappa(1-\alpha)e_G^{\alpha+\beta}] - U_F}{2A}\right\} + (W/n - w_G - \lambda e_G + \alpha\kappa e_G^{\alpha+\beta})\left\{\frac{1}{2A[w_G + \kappa(1-\alpha)e_G^{\alpha+\beta}]}\right\}$$

$$\begin{aligned}0 = {}&[-\lambda + \alpha\kappa(\alpha+\beta)e_G^{\alpha+\beta-1}]\left\{\frac{1}{2} + \frac{\ln[w_G + \kappa(1-\alpha)e_G^{\alpha+\beta}] - U_F}{2A}\right\}\\ &+ (W/n - w_G - \lambda e_G + \alpha\kappa e_G^{\alpha+\beta})\left\{\frac{\kappa(1-\alpha)(\alpha+\beta)e_G^{\alpha+\beta-1}}{2A[w_G + \kappa(1-\alpha)e_G^{\alpha+\beta}]}\right\}.\end{aligned}$$

Thus

$$\kappa(\alpha+\beta)e_G^{\alpha+\beta-1} = \lambda \Leftrightarrow e_G = \left[\frac{\kappa(\alpha+\beta)}{\lambda}\right]^{\frac{1}{1-\alpha-\beta}}. \tag{A.2}$$

PROPOSITION 3. *If revolution is a threat, human capital is likely to decrease in natural resources. It will decrease for sure if there is sufficient uncertainty in the electoral process (high A) or if uneducated citizens are reasonably good at managing natural resources (high $\gamma(0)$) or if the opposition is very weak ($\delta(e_F)$ low and with a low upper bound).*

Proof. Since the no-revolution constraint binds, the government maximises:

$$\max_{e_G, w_G}(W/n - w_G - \lambda e_G + \alpha\kappa e_G^{\alpha+\beta}) \times \min\left(\max\left\{0, \frac{1}{2} + \frac{\ln[w_G + \kappa(1-\alpha)e_G^{\alpha+\beta}] - U_F}{2A}\right\}, 1\right)$$

subject to

$$w_G + \kappa(1-\alpha)e_G^{\alpha+\beta} \geq \gamma(e_G)R(W/n)\exp(-c/q), \tag{A.3}$$

or equivalently,

$$\max_{e_G}\left[W/n - \gamma(e_G)R(W/n)\exp(-c/q) + \kappa e_G^{\alpha+\beta} - \lambda e_G\right] \times \min\left\{\max\left[0, \frac{1}{2} + \frac{\ln\gamma(e_G) + \ln R(W/n) - c/q - U_F}{2A}\right], 1\right\}.$$

The first order conditions are[1]

$$\begin{aligned} G'(e_G) \equiv & \left\{-R(W/n)[\gamma'(e_G)\exp(-c/q)] + \kappa(\alpha+\beta)e_G^{\alpha+\beta-1} - \lambda\right\} \\ & \times \left[\frac{1}{2} + \frac{\ln\gamma(e_G) + \ln R(W/n) - c/q - U_F}{2A}\right] \\ & + \left[\frac{\gamma'(e_G)}{2A\gamma(e_G)}\right]\left[W/n - \gamma(e_G)R(W/n)\exp(-c/q) + \kappa e_G^{\alpha+\beta} - \lambda e_G\right] \\ & = 0, \end{aligned}$$

which implies that

$$\begin{aligned} \frac{\partial e_G}{\partial W/n} = & \frac{[\gamma'(e_G)\exp(-c/q)]}{G''(e_G)}\left[\frac{1}{2} + \frac{\ln\gamma(e_G) + \ln R(W/n) - c/q - U_F}{2A}\right] \\ & - \frac{1}{G''(e_G)}\left[\frac{1}{2AR(W/n)} - \frac{\partial U_F}{\partial W/n}\frac{1}{2A}\right] \\ & \times \left\{-R(W/n)\left[\gamma'(e_G)\exp\left(-\frac{c}{q}\right)\right] + \kappa(\alpha+\beta)e_G^{\alpha+\beta-1} - \lambda\right\} \\ & - \frac{[1 - \gamma(e_G)\exp(-c/q)]}{G''(e_G)}\left[\frac{\gamma'(e_G)}{2A\gamma(e_G)}\right], \end{aligned}$$

where

$$\frac{\partial U_F}{\partial W/n} = \frac{\delta(e_F)}{\delta(e_F)W/n - \lambda e_F + \kappa e_F^{\alpha+\beta}},$$

so that

[1] When calculating the first order conditions we implicitly assume that $1/2+(U_G-U_F)/A < 1$. If the expression becomes bigger than 1, the government wins the elections for sure. In this case it only has to take the no-revolution constraint into account.

$$\frac{\partial e_G}{\partial W/n} = \frac{[\gamma'(e_G)\exp(-c/q)]}{G''(e_G)}\left[\frac{1}{2} + \frac{\ln\gamma(e_G) + \ln R(W/n) - c/q - U_F}{2A}\right] \tag{A.4}$$

$$-\frac{1}{G''(e_G)}\left[\frac{1}{2AR(W/n)} - \frac{\delta(e_F)}{\delta(e_F)W/n - \lambda e_F + \kappa e_F^{\alpha+\beta}}\frac{1}{2A}\right] \tag{A.5}$$

$$\times\left\{-R(W/n)\left[\gamma'(e_G)\exp\left(-\frac{c}{q}\right)\right] + \kappa(\alpha+\beta)e_G^{\alpha+\beta-1} - \lambda\right\} \tag{A.6}$$

$$-\frac{[1-\gamma(e_G)\exp(-c/q)]}{G''(e_G)}\left[\frac{\gamma'(e_G)}{2A\gamma(e_G)}\right]. \tag{A.7}$$

In order to explain how changes in natural resources affect the induced level of human capital when the no-revolution constraint is binding it remains to sign $\partial e_G/\partial W/n$. We know that: $G''(e_G) \leq 0$ to guarantee the satisfaction of second order conditions. Also, $(1/2 + (\ln\gamma(e_G) + \ln R(W/n) - c/q - U_F)/2A) \geq 0$, since it is a probability. Hence the first line of the expression (A.4) is negative.

Claim 1. $\{-R(W/n)[\gamma'(e_G)\exp(-c/q)] + \kappa(\alpha+\beta)e_G^{\alpha+\beta-1} - \lambda\} < 0.$

Proof.

$$\begin{aligned}
&G'(e_G)\left\{-R(W/n)\left[\gamma'(e_G)\exp\left(-\frac{c}{q}\right)\right] + \kappa(\alpha+\beta)e_G^{\alpha+\beta-1} - \lambda\right\}\\
&\times\left[\frac{1}{2} + \frac{\ln\gamma(e_G) + \ln R(W/n) - c/q - U_F}{2A}\right]\\
&+\left[\frac{\gamma'(e_G)}{2A\gamma(e_G)}\right]\left[W/n - \gamma(e_G)R(W/n)\exp\left(-\frac{c}{q}\right) + \kappa e_G^{\alpha+\beta} - \lambda e_G\right]\\
&= 0,
\end{aligned}$$

but $\{\gamma'(e_G)/[2A\gamma(e_G)]\}[W/n - \gamma(e_G)R(W/n)\exp(-c/q) + \kappa e_G^{\alpha+\beta} - \lambda e_G] > 0$, since $[W/n - \gamma(e_G)\times R(W/n)\exp(-c/q) + \kappa e_G^{\alpha+\beta} - \lambda e_G]$ are profits of incumbent and $\gamma'(e_G) > 0$, and the result follows.

Claim 2. *For*

$$\delta(e_F) < \delta_{\max} = \frac{\underline{\delta}W/n}{W/n + \kappa(\alpha\kappa/\lambda)^{\frac{\alpha+\beta}{1-\alpha-\beta}} - \lambda(\alpha\kappa/\lambda)^{\frac{1}{1-\alpha-\beta}}},$$

we have that

$$\left[\frac{1}{2AR(W/n)} - \frac{\delta(e_F)}{\delta(e_F)W/n - \lambda e_F + \kappa e_F^{\alpha+\beta}}\frac{1}{2A}\right] > 0. \tag{A.8}$$

Proof. By definition.

$$R(W/n) = W/n + \kappa(1-\alpha)e_R^{\alpha+\beta}.$$

In the case of revolution there is no subsidy to human capital so that

$$\begin{aligned}
\kappa(1-\alpha)e_R^{\alpha+\beta} &= \omega - \lambda e_R\\
&= \kappa e_R^{\alpha+\beta} - \lambda e_R.
\end{aligned}$$

So (A.5) becomes

$$\frac{1}{W/n+\kappa e_R^{\alpha+\beta}-\lambda e_R}-\frac{\delta(e_F)}{\delta(e_F)W/n-\lambda e_F+\kappa e_F^{\alpha+\beta}}>0.$$

To show this, knowing that e_F maximises $\delta(e_F)W/n-\lambda e_F + \kappa e_F^{\alpha+\beta}$ it suffices to show

$$\frac{1}{W/n+\kappa e_R^{\alpha+\beta}-\lambda e_R}-\frac{\delta(e_F)}{\underline{\delta}W/n}>0.$$

Recall that

$$e_R=\left(\frac{\alpha\kappa}{\lambda}\right)^{\frac{1}{1-\alpha-\beta}}$$

$$\delta(e_F)<\delta_{\max}=\frac{\underline{\delta}W/n}{W/n+\kappa(\alpha\kappa/\lambda)^{\frac{\alpha+\beta}{1-\alpha-\beta}}-\lambda(\alpha\kappa/\lambda)^{\frac{1}{1-\alpha-\beta}}}.$$

Claims (1) and (2) show that the second/third line of (A.4) is also negative. Unfortunately the fourth line is unambiguously positive since $[1-\gamma(e_G)\ \exp\ (-c/q)]>0$ and $\gamma'(e_G)>0$.

To proceed, let us first bound the first line of (A.4). If $\delta(e_F)$ were equal to 1, i.e. the fringe is as efficient as the government in managing natural resources, then from (5)

$$\left[\frac{(\alpha+\beta)\kappa}{\lambda}\right]^{\frac{1}{1-(\alpha+\beta)}}=e_F^*. \tag{A.9}$$

Thus

$$U_F\le\ln\left\{W/n-\lambda\left[\frac{(\alpha+\beta)\kappa}{\lambda}\right]^{\frac{1}{1-(\alpha+\beta)}}+\kappa\left[\frac{(\alpha+\beta)\kappa}{\lambda}\right]^{\frac{\alpha+\beta}{1-(\alpha+\beta)}}\right\}$$

and

$$\begin{aligned}\frac{\ln\gamma(e_G)+\ln R(W/n)-c/q-U_F}{2A}&\ge\frac{1}{2A}\ln\left\{\frac{\gamma(e_G)\exp(-c/q)R(W/n)}{W/n-\lambda\left[\frac{(\alpha+\beta)\kappa}{\lambda}\right]^{\frac{1}{1-(\alpha+\beta)}}+\kappa\left[\frac{(\alpha+\beta)\kappa}{\lambda}\right]^{\frac{\alpha+\beta}{1-(\alpha+\beta)}}}\right\}\\&=\frac{1}{2A}\ln\left\{\frac{\gamma(e_G)\exp(-c/q)\left[W/n+\kappa(1-\alpha)e_R^{\alpha+\beta}\right]}{W/n-\lambda\left[\frac{(\alpha+\beta)\kappa}{\lambda}\right]^{\frac{1}{1-(\alpha+\beta)}}+\kappa\left[\frac{(\alpha+\beta)\kappa}{\lambda}\right]^{\frac{\alpha+\beta}{1-(\alpha+\beta)}}}\right\}\\&=\frac{1}{2A}\ln\left\{\frac{\gamma(e_G)\exp(-c/q)\left[W/n+\kappa(1-\alpha)(\alpha\kappa/\lambda)^{\frac{\alpha+\beta}{1-\alpha-\beta}}\right]}{W/n-\lambda\left[\frac{(\alpha+\beta)\kappa}{\lambda}\right]^{\frac{1}{1-(\alpha+\beta)}}+\kappa\left[\frac{(\alpha+\beta)\kappa}{\lambda}\right]^{\frac{\alpha+\beta}{1-(\alpha+\beta)}}}\right\}.\end{aligned}$$

Assume

$$\frac{1}{2A}\ln\left\{\frac{\gamma(0)\exp(-c/q)\left[W/n+\kappa(1-\alpha)(\alpha\kappa/\lambda)^{\frac{\alpha+\beta}{1-\alpha-\beta}}\right]}{W/n-\lambda\left[\frac{(\alpha+\beta)\kappa}{\lambda}\right]^{\frac{1}{1-(\alpha+\beta)}}+\kappa\left[\frac{(\alpha+\beta)\kappa}{\lambda}\right]^{\frac{\alpha+\beta}{1-(\alpha+\beta)}}}\right\}\ge-\frac{1}{4} \tag{A.10}$$

$$\frac{1}{2A}\left[\ln\gamma(0)-\frac{c}{q}\right]+\frac{1}{2A}\ln\left\{\frac{W/n+\kappa(1-\alpha)(\alpha\kappa/\lambda)^{\frac{\alpha+\beta}{1-\alpha-\beta}}}{W/n+(\kappa-1)\left[\frac{(\alpha+\beta)\kappa}{\lambda}\right]^{\frac{(\alpha+\beta)}{1-(\alpha+\beta)}}}\right\}\geq-\frac{1}{4}. \tag{A.11}$$

Then if we take the first and third terms of (A.4) we have

$$\frac{\partial e_G}{\partial W/n}\leq\frac{[\gamma'(e_G)\exp(-c/q)1/4]}{G''(e_G)}-\frac{1}{G''(e_G)}\left[\frac{\gamma'(e_G)}{2A\gamma(e_G)}\right]=\frac{\gamma'(e_G)}{4G''(e_G)}\left[\exp\left(-\frac{c}{q}\right)-\frac{2}{A\gamma(e_G)}\right].$$

Assume that

$$A\gamma(0)\exp\left(-\frac{c}{q}\right)>2, \tag{A.12}$$

and we are done. Both Assumptions (A.7) and (A.9) are satisfied for A sufficiently high or $\gamma(0)$ sufficiently high.

To study the case of a weak opposition, we have to distinguish two possible scenarios:

(*i*) the opposition has some chance to win the elections and
(*ii*) the government wins the election for sure.

In the first case notice that we could use the upper bound on $\delta(e_F)$ to bound the first line of (A.4). Let this bound be $\overline{\delta}$. We use it to calculate an upper bound for U_F as before. This gives us

$$U_F\leq\ln\left\{\overline{\delta}W/n-\lambda\left[\frac{(\alpha+\beta)\kappa}{\lambda}\right]^{\frac{1}{1-(\alpha+\beta)}}+\kappa\left[\frac{(\alpha+\beta)\kappa}{\lambda}\right]^{\frac{\alpha+\beta}{1-(\alpha+\beta)}}\right\}.$$

Hence by the same line of argument than before, the following assumption (A.10) would replace Assumption (A.7).

$$\frac{1}{2A}\ln\left\{\frac{\gamma(0)\exp(-c/q)\left[W/n+\kappa(1-\alpha)(\alpha\kappa/\lambda)^{\frac{\alpha+\beta}{1-\alpha-\beta}}\right]}{\overline{\delta}W/n-\lambda\left[\frac{(\alpha+\beta)\kappa}{\lambda}\right]^{\frac{1}{1-(\alpha+\beta)}}+\kappa\left[\frac{(\alpha+\beta)\kappa}{\lambda}\right]^{\frac{\alpha+\beta}{1-(\alpha+\beta)}}}\right\}\geq-\frac{1}{4} \tag{A.13}$$

$$\frac{1}{2A}\left[\ln\gamma(0)-\frac{c}{q}\right]+\frac{1}{2A}\ln\left\{\frac{W/n+\kappa(1-\alpha)(\alpha\kappa/\lambda)^{\frac{\alpha+\beta}{1-\alpha-\beta}}}{\overline{\delta}W/n+(\kappa-1)\left[\frac{(\alpha+\beta)\kappa}{\lambda}\right]^{\frac{(\alpha+\beta)}{1-(\alpha+\beta)}}}\right\}\geq-\frac{1}{4}. \tag{A.14}$$

The lower $\overline{\delta}$, the lower the necessary A to satisfy Assumption (A.10).[2]

Let us know study the case where U_F is so small that the government wins the election for sure. In this case G's problem with $U(x) = \ln(x)$ can be simplified to:

$$\max_{e_G,w_G}\left(W/n-w_G-\lambda e_G+\alpha\kappa e_G^{\alpha+\beta}\right)$$

subject to

[2] Notice that Assumption (A.13) holds for any upper bound $\overline{\delta}$ of the $\delta(\cdot)$-function, also for values bigger than 1. In these cases a higher A is needed to satisfy Assumption (A.13). However, this is not worrisome since Sub-section 1.2.1 shows that for high delta-function the no revolution constraint does not bind.

$$w_G + \kappa(1-\alpha)e_G^{\alpha+\beta} \geq \gamma(e_G)R(W/n)\exp\left(-\frac{c}{q}\right).$$

Thus an equivalent way of writing the problem is:

$$\max_{e,w} W/n - \gamma(e)R(W/n)\exp\left(-\frac{c}{q}\right) + \kappa e^{\alpha+\beta} - \lambda e.$$

The FOCs in this case are:

$$G'(e) \equiv -R(W/n)\left[\gamma'(e)\exp\left(-\frac{c}{q}\right)\right] + \kappa(\alpha+\beta)e^{\alpha+\beta-1} - \lambda = 0$$

$$\frac{\partial e}{\partial W/n} = \frac{\gamma'(e)\exp(-c/q)}{G''(e)}.$$

As before, $G''(e) \leq 0$ in the optimum, which implies that $\partial e/\partial W/n \leq 0$. This assumes that we have an interior solution. A sufficient condition for this is, $\gamma''(\hat{e}) \geq 0$ for $\hat{e}$ with $G'(\hat{e}) = 0$.