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Technical Appendix to

GOVERNMENT REVENUES AND ECONOMIC GROWTH IN WEAKLY INSTITUTIONALISED STATES

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Appendix

Form of the value functions. We now construct the citizens' value function, $V(X_{it})$, from the policy functions derived in Section 2 and show that the initial guess was correct. The guesses of all other value functions can be verified in a similar way.

Given the policy functions (9) and (11), we can write the equilibrium level of technology adoption, g_{it+1}, as a function of the parameters β, ω, γ, and χ and of the probability of a regime change, p (which itself depends on the distribution function F and on γ, ω and π):

$$g_{it+1} = g(\beta, \omega, \gamma, \chi; p).$$

Then, according to (8), the citizens' equilibrium consumption level can be written as a factor z (which again depends on β, ω, γ, χ; p) times the current firm revenue:

$$c_{it}\big|_{i\in\mathcal{C}} = z(\beta, \omega, \gamma, \chi; p) X_{it}.$$

This multiplicative expression is now used in equation (1) to derive V. Suppressing the arguments in z and g where appropriate, a citizen's value function can be expressed as

$$\begin{aligned} V &= \sum_{s=0}^{\infty} \beta^s \ln(zX_{it+s}) \\ &= \sum_{s=0}^{\infty} \beta^s \ln(X_{it+s}) + \frac{\ln(z)}{1-\beta}. \end{aligned}$$

Further, exploiting the fact that $X_{it+s} = X_{it}(1+g)^s$ yields

$$\begin{aligned} V(X_{it}) &= \frac{\ln(X_{it})}{1-\beta} + \frac{\ln(z)}{1-\beta} + \beta \ln(1+g) + \beta^2 \ln[(1+g)^2] + \cdots \\ &= \frac{\ln(X_{it})}{1-\beta} + \underbrace{\frac{\ln(z)}{1-\beta} + \beta \frac{\ln(1+g)}{(1-\beta)^2}}_{\equiv A}, \end{aligned}$$

and it can easily be verified that $V(X_{it})$ satisfies the functional equation representing the recursive formulation of the citizens' decision problem in Section 2.

Solving the citizens' optimisation problem under democracy. The first step is to guess the form of the citizens' value function under democracy, $V^D(X_{it})$. So suppose that the value function is of the form $\ln(X_{it})(1-\beta)^{-1} + A^D$, where A^D is a constant which depends only on the parameters of the model. Then, the set of first-order conditions is given by

$$-\frac{\chi}{1-\chi g^D_{it+1}-\sigma^D_t}+\frac{\beta}{1-\beta}\frac{1}{1+g^D_{it+1}}-\eta_1=0 \tag{A.1}$$

$$-\frac{1}{1-\chi g^D_{it+1}-\sigma^D_t}+\frac{\eta_1}{\chi}-\eta_2(\omega+\gamma)=0 \tag{A.2}$$

$$\eta_1\left(g^D_{it+1}-\sigma^D_t/\chi\right)=0, \eta_2\left(\sigma^D_t(\omega+\gamma)-\gamma\right)=0, \eta_1\geq 0, \text{ and } \eta_2\geq 0$$

where η_1 and η_2 are the Lagrange multipliers associated with the two inequality constraints.

To solve the above system, I establish in a first step that the constraint $g^D_{it+1}-\sigma^D_t/\chi\leq 0$ always binds in optimum. Suppose it did not. Then, $\eta_1=0$ and (A.2) would imply $\eta_2<0$ – which violates the condition $\eta_2\geq 0$ stated above.

The next step is now to use $g^D_{it+1}=\sigma^D_t/\chi$ in (A.1) and (A.2) in order to substitute for g^D_{it+1}. Then, adding the two resulting expressions together yields

$$-\frac{2\chi}{1-2\sigma^D_t}+\frac{\beta}{1-\beta}\frac{1}{1+\sigma^D_t/\chi}-\chi\eta_2(\omega+\gamma)=0,$$

from which (12) immediately follows. The final step is to verify that the guess of the value function was indeed correct. This can be done along the lines demonstrated above.

Proof of Lemma 1. Note first that the monotone hazard rate assumption imposed on the distribution function *F(x)* carriers over to *ρH(x)*. The strict quasi-concavity of $\vartheta(\gamma)$ can then immediately be seen by looking at the first derivative of the function,

$$\frac{\mathrm{d}\vartheta}{\mathrm{d}\gamma}=\frac{\beta}{\omega}\{1-\rho H[(\gamma/\omega)\Pi]\}\left\{\frac{\omega^2}{(\omega+\gamma)^2}-\frac{\rho h[(\gamma/\omega)\Pi]}{1-\rho H[(\gamma/\omega)\Pi]}\left(\frac{\gamma}{\omega+\gamma}+\chi\right)\Pi\right\},$$

which – as γ increases – may change its sign only once, namely from positive to negative.

Regarding the second claim, note that $\beta[\gamma(\omega+\gamma)^{-1}+\chi]-\chi$ is strictly positive if γ is sufficiently close to ω (due to Assumption A1). So, by choosing ρ arbitrarily close to 0, we can always ensure that $\vartheta(\gamma^*)=\max\limits_{\gamma\in[0,\omega]}\left\{\beta_{\mathcal{RE}}\left[\gamma(\omega+\gamma)^{-1}+\chi\right]-\chi\right\}$ is strictly positive.

Proof of Proposition 3. Suppose first that $\gamma^*\in(0,\omega)$. Then, since the monotone hazard rate condition holds, it follows from the first-order condition,

$$\frac{\mathrm{d}\vartheta(\gamma^*)}{\mathrm{d}\gamma}=0\Leftrightarrow\left\{\frac{1}{(1+\gamma^*/\omega)^2}-\frac{\rho h[(\gamma^*/\omega)\Pi]}{1-\rho H[(\gamma^*/\omega)\Pi]}\left(\frac{\gamma^*/\omega}{1+\gamma^*/\omega}+\chi\right)\Pi\right\}=0,$$

that $\partial\gamma^*/\partial\rho$ and $\partial\gamma^*/\partial\chi$ are strictly negative and that $\partial\gamma^*/\partial\omega$ is strictly positive.

Consider now the corner solution $\gamma^*=\omega$. With $\mathrm{d}\vartheta/\mathrm{d}\gamma|_{\gamma=\omega}>0$, marginal increases in ρ or χ leave γ^* unaffected since $\arg\max\vartheta(\gamma)>\omega$. Yet, since $\arg\max\vartheta(\gamma)$ falls as ρ and χ increase, the case $\gamma^*<\omega$ becomes relevant at some point. Finally, if $\gamma^*=\omega$, we have $\partial\gamma^*/\partial\omega=1>0$.