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Technical Appendix to
FINANCIAL SYSTEM ARCHITECTURE AND THE CO-EVOLUTION OF BANKS AND CAPITAL MARKETS

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Appendix

Define

- $y(x) \equiv \dfrac{\lambda(x)}{\lambda'(x)} - x$, and denote y^{-1} as the inverse function of y;[1]
- $\bar{x} \equiv \dfrac{pk_1}{pk_1 + (1-p)(1-k_1)}$, with k_1 being defined such that

$$1 + Z = \frac{1 + (1-k_1)(\bar{\omega}/\bar{N})y^{-1}\left[\dfrac{\bar{N}}{(1-k_1)\bar{\omega}}\right]}{k_1\lambda\left\{y^{-1}\left[\dfrac{\bar{N}}{(1-k_1)\bar{\omega}}\right]\right\}};$$

- k_2, such that $\dfrac{1 + (1-k_2)(\bar{\omega}/\bar{N})y^{-1}\left[\dfrac{\bar{N}}{(1-k_2)\bar{\omega}}\right]}{k_2\lambda\left\{y^{-1}\left[\dfrac{\bar{N}}{(1-k_2)\bar{\omega}}\right]\right\}} = H - Z/p.$

Parametric Restriction 1:

$$Z < \bar{Z} \equiv p\left(X - \frac{1 + (1-\bar{x})(\bar{\omega}/\bar{N})y^{-1}\left[\dfrac{\bar{N}}{(1-\bar{x})\bar{\omega}}\right]}{\bar{x}\lambda\left\{y^{-1}\left[\dfrac{\bar{N}}{(1-\bar{x})\bar{\omega}}\right]\right\}}\right). \tag{A1}$$

Parametric Restriction 2:

$$\tau \in (\underline{\tau}, \bar{\tau}), \tag{A2}$$

where

[1] It is straightforward to check that $y' > 0$ and hence y is invertible.

$$\underline{\tau} \equiv \frac{\bar{x}Z + p\left(\dfrac{1+(1-\bar{x})(\bar{\omega}/\bar{N})y^{-1}\left[\dfrac{\bar{N}}{(1-\bar{x})\bar{\omega}}\right]}{\lambda\left\{y^{-1}\left[\dfrac{\bar{N}}{(1-\bar{x})\bar{\omega}}\right]\right\}} - \dfrac{(1-\bar{x})(\bar{\omega}/\bar{N})y^{-1}\left[\dfrac{\bar{N}E}{(1-\bar{x})\bar{\omega}}\right]+E}{\lambda\left\{y^{-1}\left[\dfrac{\bar{N}E}{(1-\bar{x})\bar{\omega}}\right]\right\}}\right)}{p(1-E)} - \bar{x},$$

$$\bar{\tau} \equiv \frac{k_2 X - \dfrac{(1-k_2)(\bar{\omega}/\bar{N})y^{-1}\left[\dfrac{\bar{N}E}{(1-k_2)\bar{\omega}}\right]+E}{\lambda\left\{y^{-1}\left[\dfrac{\bar{N}E}{(1-k_2)\bar{\omega}}\right]\right\}}}{1-E} - k_2.$$

Proof of Lemma 1. For direct market financing, it is clear that in equilibrium the borrower's repayment obligation with a non-intermediated debt is stipulated such that the expected payoff to the lending investor just equals 1, the financing provided: if the lender's participation constraint were not binding, the borrower would borrow from another investor charging a lower debt repayment. The same argument applies to securitised debt. Consider relationship lending. Note that financial intermediation here has a constant-returns-to-scale technology and each bank deals with only one borrower. Now, it is clear that the participation constraints for depositors and investors in the capital market (who provide equity capital to the bank) will be binding in equilibrium. If the bank were to obtain funds at rates that slackened these participation constraints, the borrower would be better off going to a bank that procured less expensive financing. Moreover, the bank loan repayment, L, will also be stipulated such that the bank's expected payoff just covers its cost (deposit gathering and screening): if this were not the case, the borrower would opt for another bank charging a lower interest rate (lower L).

Proof of Lemma 2. In securitisation, conditional on screening (after the cost $cp^2/2$ is incurred), if $s = s_c$ and the bank were to accept the borrower, its expected payoff would be $q^C(R_{\text{sec}} - \hat{R}_{\text{sec}}) - (1-q^C)(\delta\hat{R}_{\text{sec}}) - Z - (cp^2/2)$, whereas if the bank rejects the borrower, the bank cannot recoup its screening cost and its payoff is simply $-cp^2/2$. We know from (10) and $q^C < q$ that $q^C(R_{\text{sec}} - \hat{R}_{\text{sec}}) - (1-q^C)(\delta\hat{R}_{\text{sec}}) - Z - (cp^2/2) < -cp^2/2$. Thus, in equilibrium the bank will reject the borrower if screening yields $s = s_c$. The proof for relationship lending is similar.

Proof of Lemma 3. *Direct Marketing Financing*. Substituting R_{dir} and ω into the borrower's objective function, we can rewrite the borrower's problem as:

$$\max_{\{N_{\text{dir}}\}} \pi_{\text{dir}} = X - \frac{1+(1-q)(\bar{\omega}/\bar{N})(N_{\text{dir}}/2)}{q\lambda(N_{\text{dir}})}.$$

The equilibrium investor participation, N_{dir}, is given by the following first-order-condition (FOC):

$$\frac{\lambda(N_{\text{dir}})}{\lambda'(N_{\text{dir}})} - N_{\text{dir}} = \frac{2\bar{N}}{(1-q)\bar{\omega}}. \tag{A3}$$

It is straightforward algebra to check that the second-order-condition (SOC) is satisfied. Note that the left-hand side (LHS) of (A3) is increasing in $N_{\text{dir}} : \partial\text{LHS}/\partial N_{\text{dir}} =$

$-\lambda(N_{\text{dir}})\lambda''(N_{\text{dir}})/[\lambda'(N_{\text{dir}})]^2 > 0$ since $\lambda''(N_{\text{dir}})\lambda(N_{\text{dir}}) < 0$. The right-hand-side (RHS) of (A3) is increasing in q and $\bar{N}$. Thus, we have $\partial N_{\text{dir}}/\partial q > 0$ and $\partial N_{\text{dir}}/\partial \bar{N} > 0$. The equilibrium debt repayment obligation is:

$$R_{\text{dir}} = \frac{1 + (1-q)(\bar{\omega}/\bar{N})(N_{\text{dir}}/2)}{q\lambda(N_{\text{dir}})}, \tag{A4}$$

where N_{dir} is given by (A3). Using the Envelope Theorem, we have $\partial\pi_{\text{dir}}/\partial q = -\partial R_{\text{dir}}/\partial q > 0$, and $\partial\pi_{\text{dir}}/\partial\bar{N} = -\partial R_{\text{dir}}/\partial\bar{N} > 0$.

Securitisation. Using straightforward algebra we can show from (8), (9) and (11) that:

$$R_{\text{sec}} = \hat{R}_{\text{sec}} + Z + \left(\frac{1-q}{q^A - q}\right)\left[\frac{cp^2/2}{qp + (1-q)(1-p)}\right] = \frac{1 + (1-q^A)(\bar{\omega}/\bar{N})(N_{\text{sec}}/2)}{q^A\lambda(N_{\text{sec}})} + \frac{Z}{q^A} + \frac{cp}{2q}.$$

Thus, the borrower's optimisation problem is equivalent to:

$$\min_{\{N_{\text{sec}}\}} \frac{1 + (1-q^A)(\bar{\omega}/\bar{N})(N_{\text{sec}}/2)}{q^A\lambda(N_{\text{sec}})}.$$

The equilibrium investor participation, N_{sec}, is given by the following FOC:

$$\frac{\lambda(N_{\text{sec}})}{\lambda'(N_{\text{sec}})} - N_{\text{sec}} = \frac{2\bar{N}}{(1-q^A)\bar{\omega}}. \tag{A5}$$

It can be verified that the SOC is satisfied. It is clear that the LHS of (A5) is increasing in N_{sec}, and the RHS of (A5) is increasing in q, p and $\bar{N}$. Thus, we have $\partial N_{\text{sec}}/\partial q > 0$, $\partial N_{\text{sec}}/\partial p > 0$ and $\partial N_{\text{sec}}/\partial\bar{N} > 0$. Moreover, for each fixed q, the RHS of (A5) is greater than the RHS of (A3). Thus, we have $N_{\text{sec}} > N_{\text{dir}}$, $\forall q$. The proof of the comparative statics of π_{sec} with respect to q and $\bar{N}$ is similar to that for direct market financing by using the Envelope Theorem. Finally, note that

$$\pi_{\text{sec}} = -cp^2/(2q) + p\left[X - \frac{1 + (1-q^A)(\bar{\omega}/\bar{N})(N_{\text{sec}}/2)}{q^A\lambda(N_{\text{sec}})}\right] - (Z/q)[qp + (1-q)(1-p)],$$

which is concave in p. Thus, there exists a p that maximises π_{sec}.

Relationship Loan. From the bank's and the participating investors' individual rational (IR) constraints, we have:

$$L = (1-E)\left(1 + \frac{\tau}{q^A}\right) + \frac{(1-q^A)(\bar{\omega}/\bar{N})(N_{\text{loan}}/2) + E}{q^A\lambda(N_{\text{loan}})} + \frac{cp}{2q}. \tag{A6}$$

The bank's problem can be rewritten as:

$$\min_{\{N_{\text{loan}}\}} L.$$

The equilibrium investor participation, N_{loan}, is given by the following (FOC):

$$\frac{\lambda(N_{\text{loan}})}{\lambda'(N_{\text{loan}})} - N_{\text{loan}} = \frac{2E\bar{N}}{(1-q^A)\bar{\omega}}. \tag{A7}$$

Again, it can be verified that the SOC is satisfied. The LHS of (A7) is increasing in N_{loan} and the RHS of (A7) is increasing in q, p and $\bar{N}$. Thus, we have $\partial N_{\text{loan}}/\partial q > 0$, $\partial N_{\text{loan}}/\partial p > 0$ and $\partial N_{\text{loan}}/\partial\bar{N} > 0$. The proof of the comparative statics of π_{loan} with respect to q and $\bar{N}$ is similar to that for direct market financing by using the Envelop Theorem. Finally, the existence of a payoff-maximising p follows from the same argument as in the case for securitisation by observing that π_{loan} is concave in p.

Proof of Proposition 1. *Bank Lending Scope.* Note that $L \uparrow \infty$ as $q \downarrow 0$ (see (A6)). Thus, the existence of the low cutoff, q_l, is clear based on the discussion in the text. Note that q_l is determined

as $\pi_{\text{loan}}|_{q=q_l} = 0$. Thus, $\partial q_l/\partial \bar{N} = -(\partial \pi_{\text{loan}}/\partial \bar{N})/(\partial \pi_{\text{loan}}/\partial q) < 0$, since $\partial \pi_{\text{loan}}/\partial \bar{N} > 0$ and $\partial \pi_{\text{loan}}/\partial q > 0$ (see Lemma 3).

Financing Choice. We first examine the authentic borrower's choice of funding source, assuming that the crook chooses the same funding source as the authentic borrower with the same prior credit quality q; we will show later that this indeed is a universally divine sequential equilibrium.

First, we analyse the cutoff q_h. Define q_h such that $\pi_{\text{sec}}|_{q^A=1} = \pi_{\text{dir}}|_{q=q_h}$, i.e.,

$$X - 1 - Z = X - \frac{1 + (1 - q_h)(\bar{\omega}/\bar{N})(N_{\text{dir}})}{q_h \lambda(N_{\text{dir}})}, \tag{A8}$$

where N_{dir} is given by $y(N_{\text{dir}}) \equiv \lambda(N_{\text{dir}})/\lambda'(N_{\text{dir}}) - N_{\text{dir}} = \bar{N}/[(1 - q_h)\bar{\omega}]$. The parametric assumption that Z is not too large (see (A1)) guarantees that $\pi_{\text{sec}}|_{q=q_h} > 0$ and hence securitisation is viable at $q = q_h$. It is clear that $\pi_{\text{dir}}|_{q=q_h} > \pi_{\text{sec}}|_{q=q_h}$. Note that (A8) is not a function of p. Thus, $\partial q_h/\partial p = 0$.

Next, we analyse the cutoff q_m. The existence of such a cutoff, based on the discussion in the text, can be guaranteed by the parametric assumption in (A2) that τ is neither too large nor too small. More specifically, the assumption that $\tau < \bar{\tau}$ guarantees that $\pi_{\text{sec}}|_{q=q_l} < \pi_{\text{loan}}|_{q=q_l} = 0$, and the assumption that $\tau > \underline{\tau}$ guarantees that $\pi_{\text{sec}}|_{q=q_h} > \pi_{\text{loan}}|_{q=q_h}$. Combining these with the fact that both π_{loan} and π_{sec} are increasing and concave functions of q establishes the existence of the cutoff, $q_m \in (q_l, q_h)$. To prove the comparative statics of q_m, note that q_m is determined by the following equation:

$$\pi_{\text{sec}}|_{q=q_m} - \pi_{\text{loan}}|_{q=q_m} = 0. \tag{A9}$$

We have:

$$\frac{\partial q_m}{\partial \tau} = \frac{\partial \pi_{\text{loan}}/\partial \tau}{\partial \pi_{\text{sec}}/\partial q - \partial \pi_{\text{loan}}/\partial q} < 0,$$

since a higher τ only decreases π_{loan} but not π_{sec}, thereby making securitisation a better funding choice than relationship borrowing for the authentic borrower (i.e., q_m decreases). Since $\partial \pi_{\text{loan}}/\partial \tau < 0$, we must have $\partial \pi_{\text{sec}}/\partial q - \partial \pi_{\text{loan}}/\partial q > 0$, $\forall q$. Then, we have:

$$\frac{\partial q_m}{\partial p} = -\frac{\partial \pi_{\text{sec}}/\partial p - \partial \pi_{\text{loan}}/\partial p}{\partial \pi_{\text{sec}}/\partial q - \partial \pi_{\text{loan}}/\partial q} \propto \partial \pi_{\text{loan}}/\partial p - \partial \pi_{\text{sec}}/\partial p < 0,$$

where the last inequality can be proved as follows. Note that:

$$\partial \pi_{\text{sec}}/\partial q - \partial \pi_{\text{loan}}/\partial q = (\partial \pi_{\text{sec}}/\partial q^A - \partial \pi_{\text{loan}}/\partial q^A)(\partial q^A/\partial q) > 0.$$

Thus, it must be true that $\partial \pi_{\text{sec}}/\partial q^A - \partial \pi_{\text{loan}}/\partial q^A > 0$, since $\partial q^A/\partial q > 0$. We have:

$$\partial \pi_{\text{loan}}/\partial p - \partial \pi_{\text{sec}}/\partial p = (\partial \pi_{\text{loan}}/\partial q^A - \partial \pi_{\text{sec}}/\partial q^A)(\partial q^A/\partial p) < 0,$$

since $\partial q^A/\partial p > 0$.

We now show that it is a universally divine sequential equilibrium (Banks and Sobel, 1987) that the crook will choose the same funding source as the authentic borrower with the same prior credit quality. We begin by noting that it is transparent that the equilibrium is sequential (Kreps and Wilson, 1982). Next, consider the choice between securitisation and direct market financing. For $q \in [q_m, q_h)$, the expected payoff from securitisation to an authentic borrower is π_{sec}, and the expected payoff from securitisation to a crook is $1 - p$. Consider a defection to direct capital market borrowing. Define $\mathbb{N}_d(a|$ direct market financing) as the set of investors who must participate in the capital market with direct financing to make it strictly optimal for the authentic borrower to defect from securitisation. Let $\mathbb{N}_d^o(a|$ direct market financing) be the set of investors who must participate in the capital market with direct financing to leave the authentic borrower indifferent between direct market financing and securitisation. Define N_{da}^o as:

$$\pi_{\text{sec}} = \pi_{\text{dir}}|_{N=N^o_{da}},$$

and hence

$$\mathbb{N}^o_d(a \mid \text{direct market financing}) = \{N^o_{da}\},$$

and

$$\mathbb{N}_d(a \mid \text{direct market financing}) = \{N \mid N > N^o_{da}\}.$$

Note that a crook's expected payoff from direct market financing is 1, which is strictly greater than $1 - p$. A crook only cares about the likelihood that it will get funded but not the credit terms (since it knows it will never repay the loan). Hence, if we define $\mathbb{N}_d(c \mid$ direct market financing) and $\mathbb{N}^o_d(c \mid \text{direct market financing})$ as the strict-defection and indifference sets for the crook, we have:

$$\mathbb{N}^o_d(c \mid \text{direct market financing}) = \phi,$$

and

$$\mathbb{N}_d(c \mid \text{direct market financing}) = \{N \mid N > 0\}.$$

This means

$$\mathbb{N}^o_d(a \mid \text{ direct market financing}) \cup \mathbb{N}_d(a \mid \text{ direct market financing}) \subset \mathbb{N}_d(c \mid \text{ direct market financing}).$$

Thus, by universal divinity, investors must believe that

$$\Pr(\text{defector is crook} \mid \text{defection from securitisation to direct market financing}) = 1.$$

Given this belief, neither the authentic borrower nor the crook has an incentive to defect from securitisation to direct market financing, and hence this equilibrium is universally divine.

Now, consider a defection from direct market financing to securitisation for borrowers with $q \in [q_h, 1)$. Note that a crook is strictly worse off from that defection relative to not defecting since $1 - p < 1$. An authentic borrower is also strictly worse off by defecting since π_{dir} is greater than the highest possible payoff from securitisation; note that $\pi_{\text{dir}} > \pi_{\text{sec}}|_{q^A=1} = X - 1 - Z, \forall q \in [q_h, 1)$. Thus, neither an authentic borrower nor a crook wants to defect. The other cases can be proved in a similar way.

Proof of Lemma 4. First, note that for each given p, Bertrand competition at $t = 0$ ensures that all surplus at $t = 0$ goes to the borrower regardless of its financing choice. Thus, a borrower's expected payoff at $t = -1$ (before it knows q) for each given p is given by $\int_{q_l}^{q_m} \pi_{\text{loan}} \mathrm{d}q + \int_{q_m}^{q_h} \pi_{\text{sec}} \mathrm{d}\, q + \int_{q_h}^{1} \pi_{\text{dir}} \mathrm{d}q$. Denote the value of p that maximises that payoff as p^*. Second, if a bank does not choose $p = p^*$ at $t = -1$, then it cannot attract any borrower at that time. Thus, in equilibrium every bank will choose $p = p^*$.

Proof of Proposition 2. We know from the proof of Lemma 3 (see (A6)) that $\partial L/\partial c > 0$. Hence, we have $\partial \pi_{\text{loan}}/\partial c < 0$ and, consequently, $\partial q_l/\partial c > 0$. That is, bank evolution (lower c) causes the bank to expand its lending scope from below (q_l decreases). This proves (*i*). To show (*ii*) and (*iii*), first note that the bank's optimal choice of p at $t = -1$ increases as c decreases. Then, the claim in (*ii*) follows directly from the result that $\partial q_m/\partial p < 0$ and $\partial q_h/\partial p = 0$ (see Proposition 1), and the claim in (*iii*) follows directly from the result in Lemma 3 that $\partial N_{\text{loan}}/\partial p > 0$ and $\partial N_{\text{sec}}/\partial p > 0$.

Proof of Corollary 1. If there is no securitisation, the claim that bank evolution causes the capital market to lose borrowers to the bank can be proved by observing that a lower c due to bank evolution increases the borrower's expected payoff from relationship borrowing π_{loan}, but not from direct capital financing π_{dir} (note that $\partial \pi_{\text{dir}}/\partial c = 0$). The claim that bank evolution expands the bank's lending scope from below can be established in the same way as in Proposition 2.

Proof of Proposition 3. Denote $p^* \in \operatorname{argmax}[\int_{q_l}^{q_m} \pi_{\text{loan}} \mathrm{d}q + \int_{q_m}^{q_h} \pi_{\text{sec}} \mathrm{d}q + \int_{q_h}^{1} \pi_{\text{dir}} \mathrm{d}q]$. Note that:

(i) when $\bar{N}$ increases, π_{loan} increases and, hence, q_l decreases (see Proposition 1),
(ii) $\partial \pi_{\text{loan}}/\partial p = (\partial \pi_{\text{loan}}/\partial q^A)(\partial q^A/\partial p)$.

Note that $\partial \pi_{\text{loan}}/\partial q^A$ and $\partial q^A/\partial p$ are decreasing in q. This implies that $\partial \pi_{\text{loan}}/\partial p$ is decreasing in q. Combining (i) and (ii) yields p^* being increasing in $\bar{N}$.

Proof of Corollary 2. If the bank's equity capital and its cost are exogenously fixed, then capital market evolution (larger $\bar{N}$) has no effect on the bank's rasing of equity capital from the market. Thus, π_{loan}, and hence q_l, will not change with respect to capital market evolution. Also, note that $\partial \pi_{\text{sec}}/\partial \bar{N} > 0$ (see Lemma 3). This will lead to $\partial q_m/\partial \bar{N} < 0$, causing the bank to lose some borrowers to the market.

Proof of Proposition 4. This comes from combining the results in Propositions 2 and 3.

Proof of Lemma 5. It is clear that the distribution function of $\bar{\rho}$, denoted as $F(\bar{\rho}, N)$, can be written as $F(\bar{\rho}, N) = \Pr(\bar{\rho} \le \rho) = \Pr(\rho_1 \le \rho, \ldots, \rho_N \le \rho) = \prod_{i=1}^{N} \Pr(\rho_i \le \rho) = F(\rho)^N$. Since $F(\rho) \in [0,1]$, it is clear that for any two values of N, say N_1 and N_2 with $N_1 < N_2$, $F(\bar{\rho}, N_2)$ first-order stochastic dominates $F(\bar{\rho}, N_1)$. This implies that ρ_M is increasing in N, and hence proves the lemma.

Proof of Proposition 5. Consider direct capital market financing. When the state $\{\varphi = \varphi_G, \upsilon_b = I, \upsilon_m = I\}$ occurs, i.e., both the borrower and investors perceive the project to be G, the debt repayment is R_{dir} as valued by both the borrower and investors. However, when the state $\{\varphi = \varphi_G, \upsilon_b = I, \upsilon_m = U\}$ occurs, the borrower perceives the project to be G whereas investors perceive it to be G with probability θ and B with probability $1 - \theta$. In this state, the debt repayment is R_{dir} as valued by the borrower but is only θR_{dir} as valued by investors. Thus,

$$\frac{\text{Investors' valuation of expected debt repayment}}{\text{Borrower's valuation of expected debt repayment}} = \frac{[\rho_M + (1-\rho_M)\theta](R_{\text{dir}})}{R_{\text{dir}}} = \rho_M + (1-\rho_M)\theta, \tag{A10}$$

where

$$\rho_M = N \int_0^1 F(x)^{N-1} f(x) x \mathrm{d}x = N \int_0^1 x^{N-1} x \,\mathrm{d}x = \frac{N}{1+N}. \tag{A11}$$

Thus,

$$\frac{\text{Investors' valuation of expected debt repayment}}{\text{Borrower's valuation of expected debt repayment}} \equiv \lambda(N) = \frac{\theta + N}{1+N} \in (0,1). \tag{A12}$$

The cases for securitisation and bank equity can be proved in a similar way. It is clear that $\lambda'(\cdot) > 0$ and $\lambda''(\cdot) < 0$, which is consistent with Assumption 1.

Proof of Proposition 6. Consider any borrower with prior quality q taking a relationship loan. Suppose the bank's asset-substitution moral hazard problem has been resolved, i.e., the bank only invests when $\{\varphi = \varphi_G, \upsilon_b = I\}$. From the bank's perspectives, conditional on screening yielding $s = s_a$, the net terminal payoff shared between the bank and investors is:[2]

[2] Note that: (i) $[L - (1 - E)]$ is the terminal payoff when the bank perceives the project to be worth funding, i.e., $\{\varphi = \varphi_G, \upsilon_b = I\}$, and makes a deposit repayment of $1 - E$ after receiving L from the authentic borrower; and (ii) when the bank does not invest, either because the signal is bad, i.e., $\{\varphi = \varphi_B\}$, or the signal is good but uninformative, i.e., $\{\varphi = \varphi_G, \upsilon_b = U\}$, the equity capital raised at $t = 0$ is left intact and the terminal payoff is E.

$$\Pr(\varphi = \varphi_G, \upsilon_b = I)(q^A)[L - (1 - E)] + [\Pr(\varphi = \varphi_B) + \Pr(\varphi = \varphi_G, \upsilon_b = U)](E) = \theta\mu q^A(L-1) + (1 - \theta\mu + \theta\mu q^A)(E).$$

The bank's share of the net terminal payoff, α, in equilibrium must be such that the bank's participation constraint is binding:[3]

$$[qp + (1-q)(1-p)](\alpha)[\theta\mu q^A(L-1) + (1 - \theta\mu + \theta\mu q^A)(E)] - [qp + (1-q)(1-p)]\theta\mu\tau(1-E) - cp^2/2 = 0,$$

which yields:

$$\alpha = \frac{\theta\mu\tau(1-E) + \dfrac{cp^2/2}{qp + (1-q)(1-p)}}{\theta\mu q^A(L-1) + (1 - \theta\mu + \theta\mu q^A)(E)}, \tag{A13}$$

$$L = 1 + \frac{\theta\mu\tau + \dfrac{cp^2/2}{qp + (1-q)(1-p)} - (\theta\mu\tau + \alpha - \alpha\theta\mu + \alpha\theta\mu q^A)(E)}{\alpha\theta\mu q^A}. \tag{A14}$$

If the bank invests in the negative-NPV project when the signal is good but uninformative, conditional on the borrower being authentic, with probability θ the authentic borrower is able to repay L and the net terminal payoff is $[L - (1-E)]$, and with probability $1-\theta$ the project turns out to be bad. Thus, the bank's expected payoff from investing in this negative-NPV project is

$$\alpha\theta q^A[L - (1-E)] = \frac{\theta\mu\tau + \dfrac{cp^2/2}{qp + (1-q)(1-p)} - (\theta\mu\tau + \alpha - \alpha\theta\mu)(E)}{\mu}$$

To resolve the bank's asset-substitution moral hazard problem, the regulator needs to set the bank's capital requirement high enough so that its expected payoff from investing in the negative-NPV project is no more than its expected payoff from rejecting it, which is αE, i.e.,

$$\frac{\theta\mu\tau + \dfrac{cp^2/2}{qp + (1-q)(1-p)} - (\theta\mu\tau + \alpha - \alpha\theta\mu)(E)}{\mu} \le \alpha E$$

This leads to:

$$E \ge \frac{\theta\mu\tau + \dfrac{cp^2/2}{qp + (1-q)(1-p)}}{\theta\mu\tau + \alpha(1 + \mu - \theta\mu)}. \tag{A15}$$

Substituting (A13) into (A15) and treating (A15) as an equality yields:[4]

$$E = \frac{(\theta q^A)(L-1)}{1 - \theta q^A}. \tag{A16}$$

[3] This is the bank's *ex ante* participation constraint before screening, where $\Pr(s = s_a) = qp + (1-q)(1-p)$is the probability that the bank accepts the borrower.

[4] The other solution is $E = 1$, which is dominated by the solution in (A16) because of the cost of raising bank capital. Also, note that although the capital requirement in (A16) is designed to solve the bank's asset-substitution moral hazard problem in the state $\{\varphi = \varphi_G, \upsilon_b = U\}$, it automatically solves the bank's asset-substitution moral hazard problem in the state $\{\varphi = \varphi_B\}$ as well, since the bank perceives that $\Pr(G|\varphi = \varphi_B) \le \Pr(G|\varphi = \varphi_G, \upsilon_b = U)$ and hence its project-investment distortion is less severe in the state $\{\varphi = \varphi_B\}$ than in the state $\{\varphi = \varphi_G, \upsilon_b = U\}$.

We shall now determine the equilibrium loan repayment obligation L. From the investors' perspective, the net terminal payoff to be shared with the bank is:[5]

$$q^A\left\{\begin{array}{c}\Pr(\varphi=\varphi_G,\upsilon_b=I,\upsilon_m=I)[L-(1-E)]\\ +\Pr(\varphi=\varphi_G,\upsilon_b=I,\upsilon_m=U)\theta[L-(1-E)]\end{array}\right\}+[\Pr(\varphi=\varphi_B)+\Pr(\varphi=\varphi_G,\upsilon_b=U)](E)$$
$$=\theta\mu q^A[\rho_M+(1-\rho_M)\theta](L-1)+\{1-\theta\mu+\theta\mu q^A[\rho_M+(1-\rho_M)\theta]\}(E).$$

The ownership sharing, α, is determined such that the individual rationality (IR) constraint for the participating investors is binding in equilibrium, i.e.,

$$(1-\alpha)\left(\begin{array}{c}\theta\mu q^A[\rho_M+(1-\rho_M)\theta](L-1)\\ +\{1-\theta\mu+\theta\mu q^A[\rho_M+(1-\rho_M)\theta]\}(E)\end{array}\right)-(1-q^A)(\omega/2)=E, \tag{A17}$$

where $\omega=(\bar{\omega}/\bar{N})(N_{\text{loan}})$. Combining (A13) and (A17), and substituting E with (A16), we have:

$$L=1+\left(\frac{1-\theta q^A}{\theta q^A}\right)\left[\frac{\dfrac{\theta\mu\tau}{1+\mu-\theta\mu}+\dfrac{(1-q^A)(\bar{\omega}/\bar{N})(N_{\text{loan}}/2)}{1+\rho_M(1-\theta)\mu}}{\dfrac{\theta\mu\tau}{1+\mu-\theta\mu}+\dfrac{\rho_M(1-\theta)\mu}{1+\rho_M(1-\theta)\mu}}\right]. \tag{A18}$$

Substituting (A18) into (A16) yields the regulatory capital requirement:

$$E=\frac{\dfrac{\theta\mu\tau}{1+\mu-\theta\mu}+\dfrac{(1-q^A)(\bar{\omega}/\bar{N})(N_{\text{loan}})}{1+\rho_M(1-\theta)\mu}}{\dfrac{\theta\mu\tau}{1+\mu-\theta\mu}+\dfrac{\rho_M(1-\theta)\mu}{1+\rho_M(1-\theta)\mu}}. \tag{A19}$$

Note that the equilibrium capital market agreement parameter when there are N_{loan} investors participating in the capital market is given by (see the proof of Proposition 5):

$$\rho_M=\frac{N_{\text{loan}}}{1+N_{\text{loan}}}. \tag{A20}$$

Substituting (A20) into (A18) and (A19) yields:

$$L=1+\left(\frac{1-\theta q^A}{\theta q^A}\right)\left(\frac{\Delta_3}{\Delta_1}\right)\left(\frac{N_{\text{loan}}^2+\Delta_2 N_{\text{loan}}+\Delta_1}{N_{\text{loan}}+\Delta_3}\right), \tag{A21}$$

$$E=\left(\frac{\Delta_3}{\Delta_1}\right)\left(\frac{N_{\text{loan}}^2+\Delta_2 N_{\text{loan}}+\Delta_1}{N_{\text{loan}}+\Delta_3}\right), \tag{A22}$$

[5] First, investors perceive the net terminal payoff to be $[L-(1-E)]$ when they agree with the bank that the authentic borrower's project is worth funding, i.e., $\{\varphi=\varphi_G,\ \upsilon_b=I,\ \upsilon_m=I\}$. Second, when the bank thinks the project is worth funding but investors disagree, i.e., $\{\varphi=\varphi_G,\ \upsilon_b=I,\ \upsilon_m=U\}$, the investment decision rests entirely with the bank, so investors perceive that with probability θ the authentic borrower is able to repay L, and the net terminal payoff is $[L-(1-E)]$, and with probability $1-\theta$ the authentic borrower's project defaults and nothing is left to share. Third, when the bank does not invest, i.e., $\{\varphi=\varphi_B\}$ and $\{\varphi=\varphi_G,\ \upsilon_b=U\}$, the equity capital raised is left intact and the net terminal payoff is E.

where

$$\Delta_1 \equiv \frac{4\mu\tau\theta\bar{N}}{\bar{\omega}(1-q^A)(1+\mu-\theta\mu)},$$

$$\Delta_2 \equiv 2 + \frac{2\mu\tau\theta[1+(1-\theta)\mu]\bar{N}}{\bar{\omega}(1-q^A)(1+\mu-\theta\mu)},$$

$$\Delta_3 \equiv \frac{2\tau\theta}{(1+\mu-\theta\mu)(1-\theta+\theta\tau)}.$$

What remains to be determined is the equilibrium investor participation N_{loan} for relationship lending. The bank chooses L to maximise the authentic borrower's payoff from relationship borrowing, by minimising L, given by (A21). It is easy to show that L is a convex function of N_{loan}. Thus, the first-order-condition (FOC) for optimality yields the solution for N_{loan} given by:

$$N_{\text{loan}} = \sqrt{\Delta_1 - \Delta_2\Delta_3 + \Delta_3^2} - \Delta_3. \tag{A23}$$

Substituting (A23) into (A22) gives E. The claim that $\partial E/\partial q < 0$ can be proved.