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EQUILIBRIUM SEARCH WITH TIME-VARYING UNEMPLOYMENT BENEFITS

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Appendix

A. Derivation of (12) to (14)

We start by deriving (12). From (9), when $\pi(w_s) = 0$,

$$w_s = \frac{m(\theta)(1-\gamma)(x-c) - c\theta(r+\delta)}{m(\theta)(1-\gamma)}.$$

Equating this to the expression for w_s given by (5) for the case of risk neutrality gives,

$$\frac{m(\theta)(1-\gamma)(x-c) - c\theta(r+\delta)}{m(\theta)(1-\gamma)} = w_b + \frac{(r+\delta)(w_b - b - h)}{\lambda}.$$

Using (8) with $\pi(w_b) = 0$,

$$m(\theta)(x-c) = m(\theta)w_b + c\theta(r+\delta).$$

Substitution then gives

$$\frac{-\gamma c\theta(r+\delta)}{m(\theta)(1-\gamma)} = \frac{(r+\delta)(w_b - b - h)}{\lambda}.$$

Thus,

$$w_b - b - h = \frac{-\gamma\lambda c\theta}{m(\theta)(1-\gamma)} = \frac{-[m(\theta)/\lambda + m(\theta)]\lambda c\theta}{m(\theta)[\lambda/\lambda + m(\theta)]} = -c\theta,$$

which verifies (12).

Next, set the expression for w_b from (12) equal to the one given by (6) with risk neutrality; that is,

$$b + h - c\theta = \frac{[r + \phi m(\theta)](b+h) + \lambda(s+h)}{r + \lambda + \phi m(\theta)}.$$

Solving for ϕ verifies (13).

Finally, from (8)

$$c\theta(r+\delta) = m(\theta)(x - w_b - c).$$

That is,

$$c\theta(r+\delta) = m(\theta)(x - b - h + c\theta - c),$$

which, after rearrangement, verifies (14).

B. Comparative Statics Derivations

(a) Comparative statics for θ: Using (14),

$$\frac{\partial\theta}{\partial b} = \frac{m(\theta)}{c[m(\theta) - (r+\delta)] + m'(\theta)[c\theta + (x - c - b - h)]}.$$

The denominator of this expression is positive since, from (14), we have

$$c[m(\theta) - (r+\delta)] = -[m(\theta)/\theta](x - c - b - h) > 0$$

and

$$c\theta + (x - c - b - h) = c\theta(r+\delta)/m(\theta) > 0.$$

Thus, $\partial\theta/\partial b > 0$. Since neither s nor λ enters into (14), we have $\partial\theta/\partial s = \partial\theta/\partial\lambda = 0$.

(b) Comparative statics for w_b: Using (12), $\partial w_b/\partial b = 1 - c(\partial\theta/\partial b)$. That is,

$$\frac{\partial w_b}{\partial b} = \frac{-c(r+\delta) + m'(\theta)[c\theta + (x - c - b - h)]}{c[m(\theta) - (r+\delta)] + m'(\theta)[c\theta + (x - c - b - h)]}.$$

From (14), $c(r+\delta) = (m(\theta)/\theta)[c\theta + (x - c - b - h)]$, so by substitution

$$\frac{\partial w_b}{\partial b} = \frac{[(-m(\theta)/\theta) + m'(\theta)][c\theta + (x - c - b - h)]}{c[m(\theta) - (r+\delta)] + m'(\theta)[c\theta + (x - c - b - h)]}.$$

As $m'(\theta)\theta < m(\theta)$ and $c\theta + (x - c - b - h) > 0$ we have $\partial w_b/\partial b < 0$. Since s and λ appear in neither (12) nor (14) it follows that $\partial w_b/\partial s = \partial w_b/\partial\lambda = 0$.

(c) Comparative statics for w_s: From (5′),

$$\frac{\partial w_s}{\partial b} = \left(\frac{r+\delta+\lambda}{\lambda}\right)\frac{\partial w_b}{\partial b} - \frac{r+\delta}{\lambda} < 0 \text{ and}$$
$$\frac{\partial w_s}{\partial\lambda} = \frac{(b + h - w_b)(r+\delta)}{\lambda^2} = \frac{c\theta(r+\delta)}{\lambda^2} > 0.$$

Finally, $\partial w_s/\partial s = 0$.

(d) Comparative statics for ϕ: From (13),

$$\frac{\partial\phi}{\partial b} = \frac{\lambda}{c\theta m(\theta)} + \frac{\partial\phi}{\partial\theta}\frac{\partial\theta}{\partial b},$$

where

$$\frac{\partial\phi}{\partial\theta} = \frac{c\theta m(\theta)[-c(r+\lambda)] - [\lambda(b-s) - c\theta(r+\lambda)][cm(\theta) + c\theta m'(\theta)]}{[c\theta m(\theta)]^2}$$

and

$$\frac{\partial\theta}{\partial b} = \frac{m(\theta)}{c[m(\theta) - (r+\delta)] + m'(\theta)[c\theta + (x - c - b - h)]}.$$

We thus have

$$\frac{\partial\phi}{\partial b} = \frac{\lambda}{c\theta m(\theta)} - \frac{m(\theta)\lambda(b-s) + \theta m'(\theta)[\lambda(b-s) - c\theta(r+\lambda)]}{c\theta^2 m(\theta)\{c[m(\theta) - (r+\delta)] + m'(\theta)[c\theta + (x - c - b - h)]\}}.$$

Multiplying both sides by $c\theta m(\theta)$, the sign of $\partial\phi/\partial b$ is the same as that of

$$\lambda - \frac{m(\theta)\lambda(b-s) + \theta m'(\theta)[\lambda(b-s) - c\theta(r+\lambda)]}{c\theta[m(\theta)-(r+\delta)] + m'(\theta)\theta[c\theta + (x-c-b-h)]}.$$

Since the denominator of the fraction is positive, the sign of $\partial\phi/\partial b$ is the same as that of

$$\lambda c\theta[m(\theta)-(r+\delta)] + \lambda m'(\theta)\theta[c\theta + (x-c-b-h)]$$
$$- m(\theta)\lambda(b-s) - \theta m'(\theta)[\lambda(b-s) - c\theta(r+\lambda)].$$

From (14), $x - c - b - h = [c\theta(r+\delta)/m(\theta)] - c\theta$; thus, the sign of $\partial\phi/\partial b$ is the same as that of

$$\lambda m(\theta)[c\theta - (b-s)] - \lambda c\theta(r+\delta)\left[1 - \frac{m'(\theta)\theta}{m(\theta)}\right] - \theta m'(\theta)[\lambda(b-s) - c\theta(r+\lambda)].$$

The first and third of these three terms are negative by (13); specifically, by the condition $\phi > 0$. The second term is negative by $m'(\theta)\theta < m(\theta)$. We thus have $\partial\phi/\partial b < 0$.

Next,

$$\frac{\partial\phi}{\partial\lambda} = \frac{b-s-c\theta}{c\theta m(\theta)} > 0$$

since, again from (13), $b - s - c\theta > 0$.

Finally,

$$\frac{\partial\phi}{\partial s} = -\frac{\lambda}{c\theta m(\theta)} < 0.$$

(e) Comparative statics for γ: From (10),

$$\frac{\partial\gamma}{\partial\theta} = \frac{\lambda m'(\theta)}{[\lambda + m(\theta)]^2} > 0.$$

Then $\partial\gamma/\partial\lambda = -m(\theta)/[\lambda + m(\theta)]^2 < 0$, and the rest of the derivatives of γ have the same signs as the partials of θ with respect to the various parameters. Specifically, $\partial\gamma/\partial b > 0$ and $\partial\gamma/\partial s = 0$.

(f) Comparative statics for u: From (11),

$$\frac{\partial u}{\partial b} = \frac{\partial u}{\partial\phi m(\theta)}\frac{\partial\phi m(\theta)}{\partial b} + \frac{\partial u}{\partial\gamma}\frac{\partial\gamma}{\partial b},$$

where

$$\frac{\partial u}{\partial\phi m(\theta)} = \frac{-\delta\gamma}{\{\delta + \gamma[\phi m(\theta) + \lambda]\}^2} < 0$$

and

$$\frac{\partial u}{\partial\gamma} = \frac{-\delta(\phi m(\theta) + \lambda)}{\{\delta + \gamma[\phi m(\theta) + \lambda]\}^2} < 0.$$

Let $\Psi = \delta/\{\delta + \gamma[\phi m(\theta) + \lambda]\}^2 > 0$. Then, we have

$$\frac{\partial u}{\partial b} = -\Psi\left\{\gamma\frac{\partial\phi m(\theta)}{\partial b} + [\phi m(\theta) + \lambda]\frac{\partial\gamma}{\partial b}\right\}.$$

Next from (13) we have

$$\phi m(\theta) = \frac{\lambda(b-s)}{c\theta} - (r+\lambda)$$

so

$$\frac{\partial \phi m(\theta)}{\partial b} = \frac{\lambda}{c\theta} - \frac{\lambda(b-s)}{c\theta^2}\frac{\partial\theta}{\partial b} = \frac{\lambda}{c\theta}\left[1 - \frac{(b-s)}{\theta}\frac{\partial\theta}{\partial b}\right].$$

From above,

$$\frac{\partial\gamma}{\partial b} = \frac{\lambda m'(\theta)}{[\lambda + m(\theta)]^2}\frac{\partial\theta}{\partial b} > 0.$$

Thus,

$$\frac{\partial u}{\partial b} = -\lambda\Psi\left(\frac{\gamma}{c\theta} + \frac{\partial\theta}{\partial b}\left\{\frac{m'(\theta)[\phi m(\theta)+\lambda]}{[\lambda+m(\theta)]^2} - \frac{\gamma(b-s)}{c\theta^2}\right\}\right).$$

Since

$$\frac{m'(\theta)[\phi m(\theta)+\lambda]}{[\lambda+m(\theta)]^2} = \frac{\gamma m'(\theta)[\phi m(\theta)+\lambda]}{m(\theta)[\lambda+m(\theta)]},$$

we have

$$\frac{\partial u}{\partial b} = -\frac{\lambda\gamma}{c\theta}\Psi\left(1 + \frac{\partial\theta}{\partial b}\left\{\frac{m'(\theta)[\phi m(\theta)+\lambda]c\theta}{m(\theta)[\lambda+m(\theta)]} - \frac{(b-s)}{\theta}\right\}\right).$$

Then,

$$\text{sign}\left(\frac{\partial u}{\partial b}\right) = -\text{sign}\left(1 + \frac{\partial\theta}{\partial b}\left\{\frac{m'(\theta)[\phi m(\theta)+\lambda]c\theta}{m(\theta)[\lambda+m(\theta)]} - \frac{(b-s)}{\theta}\right\}\right).$$

Without imposing more restrictions on $m(\theta)$, this latter sign is indeterminate.

Next, since $\partial\gamma/\partial s = \partial\theta/\partial s = 0$, we have

$$\frac{\partial u}{\partial s} = \frac{\partial u}{\partial\phi}\frac{\partial\phi}{\partial s}.$$

Since

$$\frac{\partial u}{\partial\phi} = \frac{-\delta\gamma m(\theta)}{\{\delta + \gamma[\phi m(\theta)+\lambda]\}^2} < 0$$

and, as shown above, $\partial\phi/\partial s < 0$, we have $\partial u/\partial s > 0$.

Finally, using $\partial\theta/\partial\lambda = 0$, we have

$$\frac{\partial u}{\partial\lambda} = \frac{\partial u}{\partial\phi}\frac{\partial\phi}{\partial\lambda} + \frac{\partial u}{\partial\gamma}\frac{\partial\gamma}{\partial\lambda} - \frac{\delta\gamma}{\{\delta + \gamma[\phi m(\theta)+\lambda]\}^2}.$$

The final term is the direct effect of λ on u. Substitution gives,

$$\begin{aligned}\frac{\partial u}{\partial\lambda} = &-\left(\frac{\delta\gamma m(\theta)}{\{\delta+\gamma[\phi m(\theta)+\lambda]\}^2}\right)\left[\frac{b-s-c\theta}{c\theta m(\theta)}\right] \\ &+ \left(\frac{\delta[\phi m(\theta)+\lambda]}{\{\delta+\gamma[\phi m(\theta)+\lambda]\}^2}\right)\left\{\frac{m(\theta)}{[\lambda+m(\theta)]^2}\right\} - \frac{\delta\gamma}{\{\delta+\gamma[\phi m(\theta)+\lambda]\}^2}.\end{aligned}$$

Or,

$$\frac{\partial u}{\partial \lambda} = -\Psi\left\{\frac{\gamma(b-s-c\theta)}{c\theta} - \frac{m(\theta)[\phi m(\theta)+\lambda]}{[\lambda+m(\theta)]^2} + \gamma\right\}.$$

The sign of $\partial u/\partial \lambda$ is the same as that of

$$-\frac{\gamma(b-s)}{c\theta} + \frac{m(\theta)[\phi m(\theta)+\lambda]}{[\lambda+m(\theta)]^2}.$$

Using (10) and (13) for γ and ϕ, the sign of $\partial u/\partial \lambda$ is the same as that of

$$\frac{-m(\theta)/\lambda + m(\theta)(b-s)}{c\theta} + \frac{m(\theta)[\lambda(b-s) - c\theta(r+\lambda)/c\theta + \lambda]}{[\lambda+m(\theta)]^2} = \frac{-m(\theta)[(b-s)m(\theta)+c\theta r]}{c\theta[\lambda+m(\theta)]^2} < 0.$$

Thus, $\partial u/\partial \lambda < 0$.