The Economic Journal, **115** (*July*), © Royal Economic Society 2005. Published by Blackwell Publishing, 9600 Garsington Road, Oxford OX4 2DQ, UK and 350 Main Street, Malden, MA 02148, USA.

PRODUCTIVITY IMPROVEMENTS IN PUBLIC ORGANISATIONS

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ECONOMIC JOURNAL, vol. 115 (*July*), pp. 671–688

Appendix

Proof of Proposition 2.

(i) Since the principal can verify the announcements of the unit managers, the only deviations that need to be checked are those below the equilibrium announcements. If one unit manager announces fewer than $\sum_i A_i$ unproductive tasks for the whole organisation, given that everybody else announces $\sum_i A_i$, his utility is $U_i = 0$ and hence smaller than utility in equilibrium. Hence, the announcement vectors $(A_i^1, \ldots, A_i^n)$, $i = 1, \ldots, n$ are a Nash equilibrium of the announcement game under the tournament scheme.

(ii) In the next step, we show the uniqueness of the Nash equilibrium in the announcement game. Suppose there is a Nash equilibrium with $\max_i(\sum_{j=1}^n \alpha_i^j) < \sum_i A_i$. For at least one unit manager, say k, the payoff in this equilibrium cannot be larger than $(1/n)[n - \max_i (\sum_{j=1}^n \alpha_i^j)]$ because all unit managers either announce an aggregate share $\sum_{j=1}^n \alpha_i^j$ or some unit managers announce fewer and therefore receive nothing. The unit manager under consideration, however, can announce $\max_{i \neq k} (\sum_{j=1}^n \alpha_i^j) + \varepsilon$ and will receive utility $U_i = n - \max (\sum_{j=1}^n \alpha_i^j) - \varepsilon$, which is larger than in the candidate equilibrium for sufficiently small ε. Hence, no other Nash equilibria of the announcement game exist.

Proof of Proposition 4. Unit managers have a strict incentive to reveal all unproductive tasks in their unit if and only if

$$\frac{\partial U_i}{\partial \alpha_i} = -\beta_{ii} + \beta'_{ii}(1 - \alpha_i) + \sum_{j \neq i} \beta'_{ij}(1 - \alpha_j) > 0.$$

Inserting β_{ii} and β_{ij} from the proposed incentive scheme yields

$$\frac{\partial U_i}{\partial \alpha_i} = -1 + \varepsilon \sum_{j \neq i} \alpha_j + \varepsilon \left(\sum_{j \neq i} 1 - \alpha_j \right) = -1 + \varepsilon(n - 1) > 0,$$

which holds if and only if $1/(n-1) < \varepsilon$ which determines the lower bound on ε.
The upper bound of ε is derived from the assumption that $A_i \in [0, \frac{1}{2}]$. The incentive scheme must guarantee that $\beta_{ii}, \beta_{ij} \in [0, 1]$. Suppose that the share of unproductive tasks in all units is $A_i = \frac{1}{2}$ $\forall i$, i.e. the share of unproductive tasks is 50% in every unit. Under the candidate incentive scheme, unit managers reveal all their unproductive tasks, i.e. $\alpha_i = \frac{1}{2}$ $\forall i$, if ε is larger than the lower bound $1/(n - 1)$. Hence, the incentive coefficients in equilibrium are given by:

$$\beta_{ii} = 1 - \varepsilon(n - 1)\frac{1}{2}, \beta_{ij} = \frac{1}{2}\varepsilon.$$

Since β_{ii}, β_{ij} must be in [0, 1] it follows immediately that $\varepsilon \leq \min[2, 2/(n-1)] = 2/(n-1)$ for $n > 1$.

Proof of Proposition 5. In order to prevent collusion the principal must provide at least some positive utility, i.e. $0 < \beta_{ik} < 1$, for the unit managers. Since $A_i \in [0, \frac{1}{2}]$ by assumption, $0 < \varepsilon < 2$ must hold. For example, suppose that $\varepsilon = 0$. Then, each unit manager $j = 2,\ldots n$, has utility $U_j = 0$ under the proposed incentive scheme if unit managers act non-cooperatively. In this scenario, unit managers can realise an allocation via the side contract $C(0,\ldots,0)$ that is weakly better for all of the unit managers. While utility remains the same for unit managers $j = 2,\ldots n$, unit manager 1 will gain through this side contract. Similar considerations hold for the upper bound of ε.

Since all unit managers receive a positive fraction of tasks in the non-cooperative equilibrium, only collusion contracts characterised by the same announcements from all unit managers need to be considered. Otherwise, unit managers would receive a utility of 0. Hence, under the discriminatory tournament scheme, a possible collusion contract would be $C(\alpha_1,\ldots,\alpha_n)$ with $\alpha_i \leq A_i$ and $\alpha_j < A_j$ for at least one j. Under such a side contract utilities are given by:

$$U_1^{Co} = n + \sum_{i=1}^{n} \alpha_i(\varepsilon\alpha_i - \varepsilon - 1)$$

$$U_j^{Co} = \frac{\varepsilon}{n-1}\sum_{i=1}^{n} \alpha_i(1-\alpha_i) < \frac{\varepsilon}{n-1}\sum_{i=1}^{n} A_i(1-A_i) = U_j^{NC}.$$

Hence, the utility of unit managers $j = 2,\ldots,n$ would be lower with such a collusion contract and therefore side contracts do not occur.

Proof of Proposition 6. We denote by $f(\alpha_i, \alpha_j)$ the absolute share of tasks unit manager i receives under a given incentive scheme when the announcements are α_i and α_j. We formulate the incentive constraints IC_i and IC_j and the conditions that managers i and j do not benefit from collusion (NC_i, NC_j).

$$\begin{aligned}
&IC_i : f(\alpha_i, \alpha_j) \geq f(\alpha_i - \Delta_i, \alpha_j)\\
&IC_j : 2 - \alpha_i - \alpha_j - f(\alpha_i, \alpha_j) \geq 2 - \alpha_i - \alpha_j + \Delta_j - f(\alpha_i, \alpha_j - \Delta_j)\\
&\quad \text{or } f(\alpha_i, \alpha_j - \Delta_j) - \Delta_j \geq f(\alpha_i, \alpha_j)\\
&NC_i : f(\alpha_i, \alpha_j) \geq f(\alpha_i - \Delta_i, \alpha_j - \Delta_j)\\
&NC_j : 2 - \alpha_i - \alpha_j - f(\alpha_i, \alpha_j) \geq 2 - \alpha_i + \Delta_i - \alpha_j + \Delta_j - f(\alpha_i - \Delta_i, \alpha_j - \Delta_j)\\
&\quad \text{or } f(\alpha_i - \Delta_i, \alpha_j - \Delta_j) - \Delta_i - \Delta_j \geq f(\alpha_i, \alpha_j).
\end{aligned}$$

The proof proceeds as follows.

The IC_j condition implies that $f(\alpha_i, \alpha_j)$ is strictly monotonically decreasing in the second argument. Since $0 \leq f(\alpha_i, \alpha_j) \leq 2$ for α_i, $\alpha_j \in [0, \frac{1}{2}]$, according to a theorem of Lebesgue (Riesz and Sz.-Nagy, 1956) $f(\alpha_i, \alpha_j)$ must be continuous almost everywhere.

Consider $f(\frac{1}{2}, \alpha_j)$ at some α_j where the function is continuous in the second argument. Let us choose some arbitrary small $\Delta_j > 0$. The IC_j implies:

$$f\left(\frac{1}{2}, \alpha_j - \Delta_j\right) \geq f\left(\frac{1}{2}, \alpha_j\right) + \Delta_j.$$

Case I: Suppose there exists some $\Delta_i^*(0 < \Delta_i^* \leq \frac{1}{2})$ such that

$$f\left(\frac{1}{2} - \Delta_i^*, \alpha_j - \Delta_j\right) = f\left(\frac{1}{2}, \alpha_j\right) + \Delta_j.$$

Then, both agents will benefit from collusion through the reduction of (Δ_i^*, Δ_j) since

$$f\left(\frac{1}{2}, \alpha_j\right) < f\left(\frac{1}{2} - \Delta_i^*, \alpha_j - \Delta_j\right)$$

and

$$f\left(\frac{1}{2} - \Delta_i^*, \alpha_j - \Delta_j\right) - \Delta_i^* - \Delta_j < f\left(\frac{1}{2}, \alpha_j\right).$$

Case II: Suppose that there exists no $\Delta_i^* > 0$ such that

$$f\left(\frac{1}{2} - \Delta_i^*, \alpha_j - \Delta_j\right) = f\left(\frac{1}{2}, \alpha_j\right) + \Delta_j.$$

As the IC_i requires that

$$f\left(\frac{1}{2}, \alpha_j - \Delta_j\right) \geq f\left(\frac{1}{2} - \Delta_i, \alpha_j - \Delta_j\right)$$

we must have

$$f\left(\frac{1}{2}, \alpha_j - \Delta_j\right) \geq f\left(\frac{1}{2} - \Delta_i, \alpha_j - \Delta_j\right) > f\left(\frac{1}{2}, \alpha_j\right) + \Delta_j \text{ for all } \Delta_i,\ \frac{1}{2} \geq \Delta_i > 0.$$

Let's choose $\Delta_i = \frac{1}{2}$. Hence, agent i benefits from collusion since

$$f\left(\frac{1}{2}, \alpha_j\right) < f(0, \alpha_j - \Delta_j).$$

Suppose agent j does not benefit from collusion. Hence

$$f(0, \alpha_j - \Delta_j) - \frac{1}{2} - \Delta_j \geq f\left(\frac{1}{2}, \alpha_j\right).$$

From IC_i $[f(\alpha_i, \alpha_j) \geq f(\alpha_i - \Delta_i, \alpha_j)]$ follows

$$f\left(\frac{1}{2}, \alpha_j - \Delta_j\right) - \frac{1}{2} - \Delta_j \geq f\left(\frac{1}{2}, \alpha_j\right).$$

Since $f(\frac{1}{2}, \alpha_j)$ is continuous at α_j this is a contradiction when Δ_j is sufficiently small. Hence, agent j also benefits from collusion which completes the proof.