The Economic Journal, **115** (*July*), © Royal Economic Society 2005. Published by Blackwell Publishing, 9600 Garsington Road, Oxford OX4 2DQ, UK and 350 Main Street, Malden, MA 02148, USA.

INSECURE PROPERTY AND TECHNOLOGICAL BACKWARDNESS

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ECONOMIC JOURNAL, vol. 115 (*July*), pp. 703–721

Appendix

Proof of Proposition 1. It follows immediately from (4) together with the corresponding optimality condition for agent 2 and the definition of p'.

Proof of Proposition 2. Using Proposition 1, together with the agents' constraints, we can write agent 1's first order conditions (3) and (4) and the corresponding conditions for agent 2 as a linear system of best responses in c_i and x_i, for $i = 1,2$. The unique solution to this linear system is given by

$$c_1 = \frac{A_2}{A_1} c_2 = \left[\frac{1}{2+\beta(1+m)}\right]\left[p + \frac{A_2}{A_1}(1-p)\right] Y, \tag{18}$$

$$x_1 = x_2 \left(\frac{A_2}{A_1}\right)^{1/(1+m)} = \left[\frac{1}{1+(A_2/A_1)^{m/(1+m)}}\right]\left[\frac{\beta m}{2+\beta(1+m)}\right]\left[p + \frac{A_2}{A_1}(1-p)\right] Y. \tag{19}$$

It is easy to verify that

$$c_1' = c_2' \left(\frac{A_2}{A_1}\right)^{m/(1+m)} = \left[\frac{1}{1+(A_1/A_2)^{m/(1+m)}}\right]\left[\frac{\beta}{2+\beta(1+m)}\right][A_1 p + A_2(1-p)] Y. \tag{20}$$

This concludes the proof.

Proof of Proposition 3. To prove the second part of the Proposition, let $U_i(A_1, A_2) \equiv \log(c_i) + \beta \log(c_i')$, where c_i and c_i' are given by (18) and (20), for $i = 1,2$. Straightforward but tedious manipulation shows that

$$\frac{\partial U_1(A_1, A_2)}{\partial A_1} > 0 \qquad \Longleftrightarrow \qquad \frac{p}{1-p} > \Psi\left(\frac{A_1}{A_2}\right), \tag{21}$$

and, by symmetry,

$$\frac{\partial U_2(A_1, A_2)}{\partial A_2} > 0 \qquad \Longleftrightarrow \qquad \frac{1-p}{p} > \Psi\left(\frac{A_2}{A_1}\right), \tag{22}$$

where

$$\Psi\left(\frac{u}{v}\right) = \frac{m + \dfrac{1+m}{\beta}\left[1 + \left(\dfrac{v}{u}\right)^{m/(1+m)}\right]}{\dfrac{u}{v} + (1+m)\left(\dfrac{u}{v}\right)^{1/(1+m)}}. \tag{23}$$

Since Ψ is decreasing, it follows that each agent's best response to any choice of technology by the other agent is to adopt either A^l or A^h. Furthermore, noting that $p \geq 1/2$, from (22) and the fact that

$$\Psi\left(\frac{A^h}{A^h}\right) = \frac{\beta m + 2(1+m)}{\beta m + 2\beta} > 1, \tag{24}$$

it follows that $\partial U_2(A^h, A^h)/\partial A_2 < 0$. Hence a small deviation from $\{A_1^*, A_2^*\} = \{A^h, A^h\}$ by agent 2 is always profitable. This proves the second part of the Proposition.

To prove the first part of the Proposition, note that $k_1 = pY - c_1 - x_1$ and $k_2 = (1-p)Y - c_2 - x_2$, and let

$$\Phi\left(\frac{u}{v}\right) = \frac{m + \frac{1}{\beta}\left[1 + \left(\frac{v}{u}\right)^{m/(1+m)}\right]}{\left(\frac{1+\beta}{\beta}\right)\frac{u}{v} + \left(\frac{1+\beta}{\beta} + m\right)\left(\frac{u}{v}\right)^{1/(1+m)}}. \tag{25}$$

Using (18) and (19), one can verify after tedious manipulation that $x_2 + c_2 \leq (1-p)Y$ if and only if $(1-p)/p \geq \Phi(A_2/A_1)$. By symmetry, $x_1 + c_1 \leq pY$ if and only if $p/(1-p) \geq \Phi(A_1/A_2)$. Noting that $\Phi > 0$, that $\lim_{(u/v)\to 1}\Phi < 1$ and that each agent's best response is to adopt either the best or the worst feasible technology, the first part of the proposition follows.

Proof of Proposition 4. Let $U_i(A_1, A_2) \equiv \log(c_i) + \beta\log(c_i')$, where c_i and c_i' are given by (18) and (20), for $i = 1,2$. We will show that

$$\{A_1^*, A_2^*\} = \begin{cases} \{A^l, A^l\} & \text{if and only if} \quad U_1(A^l, A^l) \geq U_1(A^h, A^l) \\ \{A^h, A^l\} & \text{if and only if} \quad U_1(A^l, A^l) \leq U_1(A^h, A^l) \\ \{A^l, A^h\} & \text{if and only if} \quad U_2(A^l, A^l) \leq U_2(A^l, A^h), \end{cases}$$

where $U_2(A^l, A^l) \leq U_2(A^l, A^h)$ is a sufficient condition for $U_1(A^l, A^l) \leq U_1(A^h, A^l)$.

LEMMA 1. $\{A_1^*, A_2^*\} = \{A^l, A^l\}$ *if and only if* $U_1(A^l, A^l) \geq U_1(A^h, A^l)$.

Proof. It follows from (21)–(23) that $\{A_1^*, A_2^*\} = \{A^l, A^l\}$ if and only if (*i*) $U_1(A^l, A^l) \geq U_1(A^h, A^l)$, (*ii*) $U_2(A^l, A^l) \geq U_2(A^l, A^h)$.

Hence, (*i*) is a necessary condition for $\{A_1^*, A_2^*\} = \{A^l, A^l\}$, as required. To prove the converse, use the definition of $U_i(A_1, A_2)$ to write (*i*) and (*ii*) as

$$\left(\frac{1}{2}A^l\right)^\beta \geq \left[p + \frac{A^l}{A^h}(1-p)\right]\left[\frac{A^h p + A^l(1-p)}{1 + (A^h/A^l)^{m/(1+m)}}\right]^\beta, \tag{26}$$

$$\left(\frac{1}{2}A^l\right)^\beta \geq \left[\frac{A^l}{A^h}p + (1-p)\right]\left[\frac{A^l p + A^h(1-p)}{1 + (A^h/A^l)^{m/(1+m)}}\right]^\beta, \tag{27}$$

respectively. It follows immediately that (*i*) implies (*ii*), since $p \geq 1/2$. Hence, (*i*) is both necessary and sufficient for $\{A_1^*, A_2^*\} = \{A^l, A^l\}$, as required.

LEMMA 2. $\{A_1^*, A_2^*\} = \{A^h, A^l\}$ *if and only if* $U_1(A^l, A^l) \leq U_1(A^h, A^l)$.

Proof. It follows from (21)–(23) that $\{A_1^*, A_2^*\} = \{A^h, A^l\}$ if and only if (*i*) $U_1(A^h, A^l) \geq U_1(A^l, A^l)$, (*ii*) $U_2(A^h, A^l) \geq U_2(A^h, A^h)$.

Hence, (*i*) is a necessary condition for $\{A_1^*, A_2^*\} = \{A^h, A^l\}$, as required. Next consider the converse. Recalling that $p \geq 1/2$, from (22) and the fact that, for all $A \in [A^l, A^h]$,

$$\Psi\left(\frac{A}{A}\right) = \frac{\beta m + 2(1+m)}{\beta m + 2\beta} > 1, \tag{28}$$

it follows that $\partial U_2(A_1,A_2)/\partial A_2 < 0$, for all $A_2 \leq A_1$. This implies that (*ii*) is satisfied for all values of the parameters. Thus, (*i*) is both necessary and sufficient for $\{A_1^*, A_2^*\} = \{A^h, A^l\}$, as required.

LEMMA 3. $\{A_1^*, A_2^*\} = \{A^l, A^h\}$ *if and only if* $U_2(A^l, A^h) \geq U_2(A^l, A^l)$.

Proof. It follows from (21)–(23) that $\{A_1^*, A_2^*\} = \{A^l, A^h\}$ if and only if (*i*) $U_1(A^l, A^h) \geq U_1(A^h, A^h)$, (*ii*) $U_2(A^l, A^h) \geq U_2(A^l, A^l)$.

Hence, (*ii*) is a necessary condition for $\{A_1^*, A_2^*\} = \{A^l, A^h\}$, as required. To prove the converse, suppose that $U_2(A^l, A^h) \geq U_2(A^l, A^l)$. Noting that

$$U_1(A_1, A_2) = U_2(A_1, A_2) + \left(1 + \frac{\beta m}{1+m}\right)\log\left(\frac{A_2}{A_1}\right), \tag{29}$$

and

$$U_2\left(A^h, A^h\right) = U_2\left(A^l, A^l\right) + \beta \log\left(\frac{A^h}{A^l}\right), \tag{30}$$

it follows that

$$U_1\left(A^l, A^h\right) \geq U_1\left(A^h, A^h\right) + \left(1 + \frac{\beta m}{1+m} - \beta\right)\log\left(\frac{A_h}{A_l}\right) > U_1\left(A^h, A^h\right), \tag{31}$$

and thus (*ii*) is sufficient for (*i*), as required.

It follows immediately from the proof of Lemma 1 that $U_2(A^l, A^h) \geq U_2(A^l, A^l)$ is a sufficient condition for $U_1(A^h, A^l) \geq U_1(A^l, A^l)$, as required. This concludes the proof of Proposition 4.

Proof of Proposition 5. Simply note that, for any $m > 0$, x_1 and x_2 are given by (19). It follows that $\lim_{m\to 0} x_i = 0$, for $i = 1,2$.

Proof of Proposition 6. Suppose that there is an equilibrium where $A_j = A$ and $k_j > 0$, for all *j*. It is easy to verify that it must be symmetric, since all agents start with the same wealth. Let $A_j = A$, $c_j = c$, $k_j = k$, $x_j = x$ and $c'_j = c'$ for all $j \neq i$. It is not difficult to verify that the equality of the marginal returns to production and appropriation implies that

$$\left(\frac{np'_i}{1-p'_i}\right)^{\frac{m+1}{m}} + (n-1)\left(\frac{np'_i}{1-p'_i}\right)^{\frac{1}{m}} - \frac{nA}{A_i} = 0. \tag{32}$$

This equation determines p'_i, with $np'_j = 1 - p'_i$, for all $j \neq i$. One can then solve for the unique best responses in the second stage, as a function of p'_i, A_i and $A_j = A$, just as it is done when $n = 1$. Using (32) to write A_i as a function of p'_i, one can write U_i as a function of p'_i and $A_j = A$ alone. Hence, $\partial U_i/\partial A_i = (\partial U_i/\partial p'_i)(\partial p'_i/\partial A_i)$, where $(\partial p'_i/\partial A_i) < 0$, from (32). Tedious but straightforward manipulation shows that $\partial U_i(A,\ldots,A)/\partial A_i < 0$ for all $n \geq 1$, since

$$\frac{\partial U_i(A,\ldots,A)}{\partial p'_i} = \left(\frac{n+1}{mn^2}\right)\left\{(m+n)n + \beta\left[\left(n^2-1\right) - n\right]\right\} > 0 \tag{33}$$

for all $n \geq 1$, where we have used (32) to evaluate p_i' at the efficient technology profile.

To prove the first part of the proposition, simply set $A = A^h$ and note that the previous argument implies that a small deviation from $A_i = A^h$ is always profitable. To prove the second part of the proposition, set $A = A^l$. The result follows from the previous argument, together with the continuity of $\partial U_i/\partial A_i$ in A_i. This concludes the proof.

Proof of Proposition 7. We proceed by contradiction. Using (14) and (15) one can solve for agent i's unique interior best response in the second stage, as a function of A_i, A_j and $p_{i,j}'$, where $p_{i,j}'$ must solve (14). Using (14) again to eliminate A_i one can write U_i as a function of A_j and $p_{i,j}'$ alone, in which case $\partial U_i/\partial A_i = (\partial U_i/\partial p_{i,j}')(\partial p_{i,j}'/\partial A_i)$. Since $\partial p_{i,j}'/\partial A_i < 0$, from (14), we are interested in the sign of $\partial U_i/\partial p_{i,j}'$ when $p_{i,j}' = \pi/(\pi + 1)$, since this is the unique value of $p_{i,j}' \in [1/2, 1)$ that solves (14) when $A_i = A^h$, for $i = 1,2$. The Proposition follows, after tedious manipulation, from noting that $\partial U_i/\partial p_{i,j}'$ evaluated at $p_{i,j}' = \pi/(\pi + 1)$ is continuous at $\pi = 1$, with

$$\frac{\partial U_2(A^h, \ldots, A^h)}{\partial p_{2,1}'} = 2\beta + 4\left(\frac{1+m}{m}\right)[1 - (1+\beta)(1-p)] > 0, \tag{34}$$

where $p_{2,1}'$ has been evaluated at 1/2. The inequality then follows from the fact that $1-p \leq 1/2$.