

## LEARNING TO LIKE WHAT YOU HAVE – EXPLAINING THE ENDOWMENT EFFECT

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### Appendix

*Proof of Lemma 1* To prove uniqueness suppose there exist two Nash bargaining solutions  $(x, y)$  and  $(x', y')$ . Since utility functions are strictly concave, both individuals would prefer any convex combination of  $(x, y)$  and  $(x', y')$  over  $(x, y)$  and  $(x', y')$ . These convex combinations are feasible which yields a contradiction.

*Proof of Lemma 2* If Pareto improving allocations exist, the maximised Nash product  $N(x^*, y^*)$  must be strictly positive, i.e. both individuals must be strictly better off than with their endowment. Such a Pareto improving allocation exists if the problem

$$\max_{x,y} U_2(x, y) \quad \text{s.t.} \quad F(x, y) + e_1 x = e_1$$

has a value  $U_2 > e_2$ . Let  $\tilde{x}(y)$  denote the  $x$  that solves the constraint for a given  $y$ . Implicitly differentiating we find that<sup>1</sup>

$$\frac{d\tilde{x}}{dy} = \frac{-F_y}{F_x + e_1}.$$

Substituting  $\tilde{x}(y)$  into  $U_2(x, y)$  and differentiating yields

$$\frac{dU_2}{dy} = -F_{2x} \frac{d\tilde{x}}{dy} - F_{2y} - e_2.$$

In particular, a Pareto improving allocation exists if

$$\left. \frac{dU_2}{dy} \right|_{y=0} > 0$$

or

$$F_{2x}(1, 0) \frac{F_y(1, 0)}{F_x(1, 0) + e_1} - F_{2y}(1, 0) - e_2 > 0.$$

Noting that  $\lim_{y \rightarrow 0} F_y(1, y) = +\infty$  and  $\lim_{x \rightarrow 1} F_{2x}(x, 0) = \lim_{x \rightarrow 0} F_x(x, 1) = +\infty$  implies that this condition is always fulfilled. Hence,  $U(x^*, y^*) - e_1 > 0$  and  $U_2(x^*, y^*) - e_2 > 0$ .  $\square$

*Proof of Lemma 3* At any boundary solution one of the individuals, say individual 1, receives nothing of one of the goods. Let us first look at an allocation with  $y = 0$ . In this case individual 1 is no better off than with his initial endowment, and hence such an allocation

<sup>1</sup> Recall that we have assumed that  $I$  is bounded below such that the goods always remain ‘goods’. Here, this implies  $F_x + e_1 > 0$ .

cannot be a bargaining solution by Lemma 2. Now turn to an allocation with  $x = 0$ . For this to be a bargaining solution it must hold that

$$\frac{\partial N(x, y)}{\partial x} = [F_x(0, y) + e_1][U_2(0, y) - e_2] - [U(0, y) - e_1]F_{2x}(0, y) \leq 0.$$

But since  $U_2(x^*, y^*) - e_2 > 0$  and  $\lim_{x \rightarrow 0} F_x(x, y) = +\infty$ , whereas  $F_{2x}(0, y) = F_x(1, 1 - y)$  is finite, this condition can never be fulfilled. Hence, there cannot be a boundary solution where individual 1 gets nothing. By symmetry, also all boundary solutions with individual 2 getting nothing of one of the goods can not be a solution. Hence, the solution must always be in the interior.

*Proof of Proposition 1* By Lemmata 2 and 3 for all  $e_1, e_2$  the solution is interior, and the constraints  $U(x, y) \geq e_1$  and  $U_2(x, y) \geq e_2$  are not binding. Hence,  $x^*$  and  $y^*$  are simultaneously determined by the first order conditions,  $N_x = 0, N_y = 0$ . By the implicit function theorem  $x^*(\cdot, \cdot)$  and  $y^*(\cdot, \cdot)$  are differentiable in  $e_1$  and  $e_2$ .<sup>2</sup>

We will show that for all  $e_1 \leq 0$

$$\frac{\partial F[x^*(e_1, e_2), y^*(e_1, e_2)]}{\partial e_1} = F_x \frac{\partial x^*}{\partial e_1} + F_y \frac{\partial y^*}{\partial e_1} > 0. \quad (2)$$

Differentiating  $N_x = 0, N_y = 0$  with respect to  $e_1$ , we get that

$$\frac{\partial x^*}{\partial e_1} = \frac{N_{ye_1} N_{xy} - N_{xe_1} N_{yy}}{N_{xx} N_{yy} - (N_{xy})^2} \text{ and } \frac{\partial y^*}{\partial e_1} = \frac{N_{xe_1} N_{xy} - N_{ye_1} N_{xx}}{N_{xx} N_{yy} - (N_{xy})^2}. \quad (3)$$

Since the second order necessary condition,  $N_{xx} N_{yy} - (N_{xy})^2 \geq 0$ , is satisfied at an interior solution, we will show that at  $e_1 \leq 0$

$$N_{xe_1} (F_y N_{xy} - F_x N_{yy}) + N_{ye_1} (F_x N_{xy} - F_y N_{xx}) > 0.$$

Since

$$\begin{aligned} N_{xe_1} &= F_2 - e_2 y + F_{2x}(1 - x) > 0 \\ N_{ye_1} &= (1 - x)(F_{2y} + e_2) \geq 0, \end{aligned}$$

it holds that

$$\begin{aligned} N_{xe_1} [-2F_y e_1 (F_{2y} + e_2)] + N_{ye_1} 2F_y F_{2x} e_1 &= \\ -2(F_2 - e_2 y) e_1 F_y (F_{2y} + e_2) &\geq 0. \end{aligned}$$

Note that  $F_2 > e_2 y$ . Otherwise the allocation would not be Pareto improving. Thus, it suffices to show that

$$F_y N_{xy} - F_x N_{yy} > -2F_y e_1 (F_{2y} + e_2) \quad (4)$$

and

$$F_x N_{xy} - F_y N_{xx} > 2F_y F_{2x} e_1. \quad (5)$$

At an interior solution  $MRS_1 = MRS_2$ , which implies that

<sup>2</sup> Strictly speaking, the implicit function theorem requires the second order condition  $N_{xx} N_{yy} - (N_{xy})^2$  to be non-zero. We assume this to be satisfied.

$$\frac{F_x + e_1}{F_y} = \frac{F_{2x}}{F_{2y} + e_2}.$$

Thus, we have

$$F_y F_{2x} = F_x (F_{2y} + e_2) + e_1 (F_{2y} + e_2) \quad (6)$$

Given that

$$\begin{aligned} N_{xy} &= F_{xy}(F_2 - e_2 y) + F_{2xy}(F + e_1 x - e_1) \\ &\quad - F_x(e_2 + F_{2y}) - F_y F_{2x} - e_1(F_{2y} + e_2) \end{aligned}$$

we can define

$$N' := -2F_y F_{2x} < N_{xy},$$

where the inequality follows from (6). To verify claim (4) we show that  $F_y N' - F_x N_{yy} > -2F_y e_1 (F_{2y} + e_2)$  at  $e_1 \leq 0$ .

$$\begin{aligned} F_y N' - F_x N_{yy} &= 2F_x F_y e_2 + 2F_y (F_x F_{2y} - F_y F_{2x}) - \underbrace{F_x F_{yy}(F_2 - e_2 y)}_{>0} - \underbrace{F_x F_{2yy} F}_{>0}. \end{aligned}$$

By (6) it follows that

$$2F_x F_y e_2 + 2F_y (F_x F_{2y} - F_y F_{2x}) = -2F_y e_1 (F_{2y} + e_2),$$

which proves (4).

Equation (5) is satisfied if  $F_x N' - F_y N_{xx} > 2F_y F_{2x} e_1$ . But this follows immediately:

$$\begin{aligned} F_x N' - F_y N_{xx} &= -2F_x F_y F_{2x} - F_y (F_{2xx} F - 2(F_x + e_1) F_{2x} + F_{xx}(F_2 - e_2 y)) \\ &= 2F_y F_{2x} e_1 - \underbrace{F_y F_{2xx} F}_{>0} - \underbrace{F_y F_{xx}(F_2 - e_2 y)}_{>0}. \end{aligned}$$

Therefore, fitness is strictly increasing in  $e_1$  for all  $e_1 \leq 0$ . □