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Technical Appendix to

STRATEGIC COMPLEMENTARITIES AND THE TWIN CRISES

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Appendix

Threshold Strategies

We show that if speculators act according to a threshold strategy characterised by a threshold signal θ_C, creditors will act according to a unique threshold strategy characterised by a threshold signal θ_B. The opposite case is completely analogous.

When speculators act according to threshold signal θ_C, the proportion of speculators that attack the currency at each level of the fundamentals is $G[(\theta_C - \theta)/\sigma]$. Then, given n and θ, each creditor that does not run on the bank gets $\hat{R}(\theta, n, e\{\theta, G(\theta_C - \theta)/\sigma, n\})$. After taking into account the indirect effect that θ and n have through e, this function is increasing in θ and decreasing in n. Thus, the strategic problem among creditors satisfies the conditions of the strategic problem in Morris and Shin (1998) and Morris and Shin (2003, Section 2.2). As a result, for a given θ_C, there exists a unique θ_B, for which, in (a banking-sector) equilibrium, each creditor will run on the bank if and only if she observes a signal below θ_B.

Proof of Proposition 1

Function $\theta_B(\theta_C)$ is implicitly characterised by the following equation:

$$\int_0^1 [\hat{R}(\theta_B - G^{-1}(n)\sigma, n, e\{\theta_B - G^{-1}(n)\sigma, G[G^{-1}(n) + (\theta_C - \theta_B)/\sigma], n\}) - 1]\mathrm{d}n = 0.$$

Function $\theta_B(\theta_C)$ is then increasing since $e\{\theta_B - G^{-1}(n)\sigma, n, G[G^{-1}(n) + (\theta_C - \theta_B)/\sigma]\}$ is decreasing in θ_C and increasing in θ_B, and since $\hat{R}(\theta, n, e)$ is increasing in θ and increasing in e.

Function $\theta_C(\theta_B)$ is implicitly characterised by the following equation:

$$\int_0^1 a\{\theta_C - G^{-1}(m)\sigma, m, G[G^{-1}(m) + (\theta_B - \theta_C)/\sigma]\}\mathrm{d}m = 0.$$

This function is then increasing because $G[G^{-1}(m) + (\theta_B - \theta_C)/\sigma]$ is increasing in θ_B and decreasing in θ_C, and because $a(\theta, m, n)$ is decreasing in θ and increasing in n.

Existence and Uniqueness of Threshold Equilibrium

A threshold equilibrium is characterised by two threshold signals: $\hat{\theta}_B$ and $\hat{\theta}_C$, that satisfy the following equations: $\theta_C(\hat{\theta}_B) = \hat{\theta}_C$ and $\theta_B(\hat{\theta}_C) = \hat{\theta}_B$. This implies that $\theta_B[\theta_C(\hat{\theta}_B)] = \hat{\theta}_B$.

Analysing (7) and (8) that characterise $\theta_B(\theta_C)$ and $\theta_C(\theta_B)$, respectively, we can see that $\partial\theta_B(\theta_C)/\partial\theta_C < 1$ and $\partial\theta_C(\theta_B)/\partial\theta_B < 1$. This means that $\partial\theta_B[\theta_C(\hat{\theta}_B)]/\partial\hat{\theta}_B < 1$. As a result, there is exactly one value of $\hat{\theta}_B$ that satisfies the equation $\theta_B[\theta_C(\hat{\theta}_B)] = \hat{\theta}_B$. Thus, the model has exactly one threshold equilibrium.

Broader Uniqueness

Suppose by way of negation that there is a different equilibrium – the alternative equilibrium – in which creditors run on the bank at signals above $\hat{\theta}_B$ (Note that this cannot be a threshold equilibrium.) Denote the highest signal, at which creditors switch from running to not running, in the alternative equilibrium, as $\theta_B^H(\theta_B^H > \hat{\theta}_B)$, and the highest signal, at which speculators switch from running to not running, in the alternative equilibrium, as θ_C^H. Due to the existence of upper dominance regions, θ_B^H and θ_C^H have upper bounds. The equation that characterises θ_B^H is:

$$\int_{-\infty}^{\infty} (\hat{R}\{\theta, n^A(\theta), e[\theta, m^A(\theta), n^A(\theta)]\} - 1)\frac{1}{\sigma}g[(\theta_B^H - \theta)/\sigma]\mathrm{d}\theta = 0,$$

where $n^A(\theta)$ and $m^A(\theta)$ represent the behaviour of creditors and speculators, respectively, in the alternative equilibrium. Because in the alternative equilibrium, creditors do not run on the bank at signals above θ_B^H, we know that $n^A(\theta) \leq G[(\theta_B^H - \theta)/\sigma]$. Similarly, we know that $m^A(\theta) \leq G[(\theta_C^H - \theta)/\sigma]$. Thus,

$$\int_{-\infty}^{\infty} \left(\hat{R}\left\{\theta, G\left(\frac{\theta_B^H - \theta}{\sigma}\right), e\left[\theta, G\left(\frac{\theta_C^H - \theta}{\sigma}\right), G\left(\frac{\theta_B^H - \theta}{\sigma}\right)\right]\right\} - 1\right)\frac{1}{\sigma}g\left(\frac{\theta_B^H - \theta}{\sigma}\right)\mathrm{d}\theta \leq 0.$$

Following the same steps we used in Section 2, we know this inequality is equivalent to:

$$\int_0^1 \left[\hat{R}\left(\theta_B^H - G^{-1}(n)\sigma, n, e\left\{\theta_B^H - G^{-1}(n)\sigma, G\left[G^{-1}(n) + \frac{\theta_C^H - \theta_B^H}{\sigma}\right], n\right\}\right) - 1\right]\mathrm{d}n \leq 0.$$

We compare this inequality with the equation implied by the threshold equilibrium: (7). Since $\theta_B^H > \hat{\theta}_B$, the two conditions will hold only if $\theta_B^H - \theta_C^H < \hat{\theta}_B - \hat{\theta}_C$. In particular, this means that $\theta_C^H > \hat{\theta}_C$. Now, analysing the implications of $\theta_C^H > \hat{\theta}_C$ by repeating the line of argument above, we get that this inequality requires $\theta_B^H - \theta_C^H > \hat{\theta}_B - \hat{\theta}_C$, which is of course, a contradiction. Thus, we showed that in equilibrium creditors would not run at signals above $\hat{\theta}_B$. Similarly, we can show that they would run at signals below $\hat{\theta}_B$, and that speculators would act according to the threshold $\hat{\theta}_C$.

Proof of Proposition 2

(*a*) We first show that when $\theta_B^* < \theta_C^* < \theta_B^{**} < \theta_C^{**}$, as σ approaches 0, $\hat{\theta}_B^\sigma$ and $\hat{\theta}_C^\sigma$ converge to a single value: $\hat{\theta}$ (the superscript stands for the change in equilibrium thresholds following the change in σ). Suppose, by way of negation, that when σ converges to 0, $\left|\hat{\theta}_B^\sigma - \hat{\theta}_C^\sigma\right|$ is of a higher order than σ. Then, if $(\hat{\theta}_B^\sigma - \hat{\theta}_C^\sigma)$ is positive, (7) implies that $\hat{\theta}_B^\sigma$ converges to θ_B^*. Similarly, $\hat{\theta}_C^\sigma$ converges to θ_C^{**}. As a result, $\theta_B^* > \theta_C^{**}$, which is in contradiction to the assumption that $\theta_B^* < \theta_C^* < \theta_B^{**} < \theta_C^{**}$. Similarly, if $(\hat{\theta}_B^\sigma - \hat{\theta}_C^\sigma)$ is negative, $\hat{\theta}_B^\sigma$ converges to θ_B^{**} and $\hat{\theta}_C^\sigma$ converges to θ_C^*. As a result, $\theta_C^* > \theta_B^{**}$, which is again in contradiction to the assumption that $\theta_B^* < \theta_C^* < \theta_B^{**} < \theta_C^{**}$. Thus, as σ goes to 0, the difference between $\hat{\theta}_B^\sigma$ and $\hat{\theta}_C^\sigma$ is of

an order of σ. Analysing the two equations that characterise $\hat{\theta}_B^\sigma$ and $\hat{\theta}_C^\sigma$ ($\theta_C(\hat{\theta}_B^\sigma) = \hat{\theta}_C^\sigma$ and $\theta_B(\hat{\theta}_C^\sigma) = \hat{\theta}_B^\sigma$), we can see that $\hat{\theta}_B^\sigma$ and $\hat{\theta}_C^\sigma$ are continuous in σ, and thus small changes in σ as it approaches 0 have a small effect on $\hat{\theta}_B^\sigma$ and $\hat{\theta}_C^\sigma$. Thus, we can conclude that as σ approaches 0, $\hat{\theta}_B^\sigma$ and $\hat{\theta}_C^\sigma$ converge to a single value: $\hat{\theta}$.

We now show that $\hat{\theta}_C$ must be higher than θ_C^*. The equation that characterises $\hat{\theta}_C$ is $\int_0^1 a\{\hat{\theta}_C - G^{-1}(m)\sigma, m, G[G^{-1}(m) + (\hat{\theta}_B - \hat{\theta}_C)/\sigma]\}\mathrm{d}m = 0$, and the one that characterises θ_C^* is $\int_0^1 a[\theta_C^* - G^{-1}(m)\sigma, m, 0]\mathrm{d}m = 0$. The function $a(\theta, m, n)$ is decreasing in θ, increasing in n, and strictly decreasing in θ when $a(\theta, m, n) \geq 0$. The expression $G[G^{-1}(m) + (\hat{\theta}_B - \hat{\theta}_C)/\sigma]$ is greater than or equal to 0 for all m between 0 and 1, and is strictly positive for some values of m. Thus, $\hat{\theta}_C$ must be higher than θ_C^*. Similarly, we can show that $\hat{\theta}_B$ must be lower than θ_B^{**}.

(*b*) We show that when $\theta_B^* < \theta_B^{**} < \theta_C^* < \theta_C^{**}$, as σ approaches 0, $\hat{\theta}_C^\sigma$ converges to θ_C^*. Since $\hat{\theta}_C^\sigma$ cannot be lower than θ_C^*, and since $\hat{\theta}_B^\sigma$ cannot be higher than θ_B^{**}, as σ approaches 0, $G[G^{-1}(m) + (\hat{\theta}_B - \hat{\theta}_C)/\sigma]$ converges to 0. Thus, analysing the equations that define $\hat{\theta}_C^\sigma$ and θ_C^* (see proof for part (*a*) of the Proposition), we can see that $\hat{\theta}_C^\sigma$ converges to θ_C^*. Similarly, we can show that $\hat{\theta}_B^\sigma$ converges to θ_B^{**}.