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Technical Appendix to

ON THE WELFARE EFFECTS AND POLITICAL ECONOMY OF COMPETITION-ENHANCING POLICIES

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Appendix 2

This Appendix shows that the qualitative results from the circular model can also be derived by incorporating asymmetric costs in the standard Dixit and Stiglitz (1977) model of monopolistic competition. We focus on identifying the welfare effects of increasing competition, as measured by the elasticity of substitution between products, and then briefly discuss cost reduction and entry under this specification.

Welfare Effects of Increasing Competition

There are n goods, each of which is produced by a different firm. Each consumer has the utility function $u(x_1, \ldots, x_n) = \sum_{j=1}^{n} x_j^{\alpha}$ where x_j denotes the consumption of good j, and $\alpha \in (0,1)$ is a parameter which measures the substitutability between the various goods. As in the circular model, we let proportion q of firms have *high* unit costs (c_H) and the remaining firms have *low* unit costs (c_L), where $c_H > c_L$.

We analyse symmetric Nash equilibria in which all firms with the same unit cost c_i set the same price p_i, $i \in \{H,L\}$. Taking the total number of firms to be large, and given that the other firms with unit cost c_H (respectively, c_L) charge the same price p_H (respectively, p_L), each firm i solves

$$\max_{p_i}(p_i - c_i)x_i$$

where $x_i(p_i, p_H, p_L, q)$ is the residual demand curve faced by firm i. One can show that this residual demand is given by

$$x_i = K p_i^{\frac{1}{\alpha-1}} \tag{A2.1}$$

where

$$K = \left\{\frac{n\left[q x_H^{\alpha} + (1-q)x_L^{\alpha}\right]}{w}\right\}^{\frac{1}{\alpha-1}}$$

is independent of firm i's decision, w is the wealth endowment of the representative consumer, and x_H (x_L) is the equilibrium output of the other firms that have unit cost c_H (c_L). Thus firm i chooses the price p_i that maximises $(p_i - c_i)p_i^{\frac{1}{\alpha-1}}$, yielding

$$p_i = \frac{c_i}{\alpha}. \tag{A2.2}$$

Let D_L and D_H denote equilibrium sales for low and high-cost firms, respectively. Substituting for p_i in (A2.2), and noting that $x_j = D_j$ for $j \in (L, H)$, we get:

$$D_H^{\alpha-1} = \frac{nc_H}{\alpha w}[qD_H^\alpha + (1-q)D_L^\alpha] \tag{A2.3}$$

$$D_L^{\alpha-1} = \frac{nc_L}{\alpha w}[qD_H^\alpha + (1-q)D_L^\alpha]. \tag{A2.4}$$

The selection effect on output levels can be seen by noting that $D_L/D_H = (c_H/c_L)^{\frac{1}{1-\alpha}}$. An increase in α raises the relative output level of low-cost firms in equilibrium. Dividing both sides of (A2.3) by D_H^α and (A2.4) by D_L^α, we get the equilibrium demands:

$$D_H = \frac{\alpha w}{nc_H}A^{-1}(\alpha) \tag{A2.5}$$

$$D_L = \frac{\alpha w}{nc_L}B^{-1}(\alpha) \tag{A2.6}$$

where

$$A(\alpha) = \left[q + (1-q)\left(\frac{c_H}{c_L}\right)^{\frac{\alpha}{1-\alpha}}\right] \text{ and } B(\alpha) = \left[q\left(\frac{c_L}{c_H}\right)^{\frac{\alpha}{1-\alpha}} + (1-q)\right].$$

In order to identify the selection effect on *welfare* of increasing competition, let $D_S = \alpha w/n\bar{c}$ be the equilibrium demand faced by a firm if all firms had the same unit cost equal to the average unit cost in this economy, $\bar{c} = qc_H + (1-q)c_L$.[1] Aggregate welfare is given by:

$$W = \underbrace{n\left[qD_H^\alpha + (1-q)D_L^\alpha\right]}_{\text{aggregate consumer utility}} - \underbrace{n\left[qD_H c_H + (1-q)D_L c_L\right]}_{\text{aggregate production costs}}. \tag{A2.7}$$

Using (A2.5) and (A2.6),

$$\begin{aligned} W &= n\left\{D_H^\alpha A(\alpha) - \frac{\alpha w}{n}\left[gA(\alpha)^{-1} + (1-q)B(\alpha)^{-1}\right]\right\} \\ &= n\left[\underbrace{D_S^\alpha\left(\frac{\bar{c}}{c_H}\right)^\alpha A(\alpha)^{1-\alpha}}_{\text{average consumer utility}} - \underbrace{\frac{\alpha w}{n}}_{\text{average cost}}\right] \equiv nV \end{aligned}$$

Let $F(\alpha) = D_S^\alpha$ and $\phi(\alpha, c_L, c_H) = (\bar{c}/c_H)^\alpha A(\alpha)^{1-\alpha}$. Note that $\phi(\alpha, c_L, c_H) = 1$ whenever $c_L = c_H = \bar{c}$. We then obtain the following decomposition of the total effect of a change in the degree of competition on average welfare V:

$$\frac{dV}{d\alpha} = \underbrace{\left(\frac{dF}{d\alpha} - \frac{w}{n}\right)}_{\text{expansion effect}} + \underbrace{\left\{\frac{dF}{d\alpha}[\phi(\alpha, c_L, c_H) - 1] + F\frac{d\phi}{d\alpha}\right\}}_{\text{selection effect}} \tag{A2.8}$$

where $d\phi/d\alpha > 0$. This decomposition identifies two basic effects:

(*i*) an *expansion effect*, captured by the first term on the right-hand side of (A2.8). Intensifying competition increases aggregate output, and thus consumption, abstracting from any cost asymmetry. This increase in production also raises total production costs. The first term gives the net welfare gain from this expansion.

[1] To see this, note that when $c_H = c_L = \bar{c}$, $A(\alpha) = B(\alpha) = 1$ and therefore $D_H = D_L = \alpha w/n\bar{c}$.

(*ii*) a *selection effect*, captured by the second term on the right-hand side of (A2.8). Intensifying competition raises the market share of low-cost firms which increases the aggregate consumption that can be generated for a given total cost. Thus aggregate productive efficiency increases. As in the circular model, this effect increases with the degree of cost asymmetry.

Cost Reduction

Suppose that firm i can reduce unit production cost from c_i to $c_i - e$ by incurring effort cost $g(e) = be^2$. We solve for a symmetric perfect equilibrium where all firms with initial cost parameter $c_i \in \{c_H, c_L\}$ first choose the same effort $e_i \in \{e_H, e_L\}$, and then compete in prices. From the analysis above, equilibrium profit of a firm with unit cost $c_i(e) = c_i - e$ is

$$\pi[c_i(e)] = \left(\frac{1-\alpha}{\alpha}\right)\alpha^{\frac{1}{1-\alpha}}Kc_i(e)^{\frac{\alpha}{\alpha-1}}.$$

The firm with initial cost c chooses effort level by

$$\max_e\ [\pi(c-e) - g(e)]$$

which gives the first-order condition

$$K\alpha^{\frac{1}{1-\alpha}} = 2be(c-e)^{\frac{1}{1-\alpha}}. \tag{A2.9}$$

It is easy to confirm that the second-order condition is only satisfied for effort levels on the upward sloping part of the RHS curve. The above equation thus implies that the equilibrium efforts for high and low-cost firms, e_H and e_L satisfy $e_H < e_L$, as in the circular model. This reflects a *market share effect*: high-cost firms do less cost reduction because their profit gains from doing so are proportional to their market share, and are thus smaller than those of low-cost firms.

To analyse the effect of intensifying product market competition, note that a rise in α increases the LHS of the first-order condition, and increases the slope of the RHS when $c = c_H$ by more than it increases the slope of the RHS when $c = c_L$. Thus an increase in α increases the difference $(e_L - e_H)$, as in the circular model.

Finally, we compare the laissez-faire (Nash) outcome to the second-best solution (e_H*, e_L*), whereby the 'social planner' chooses the effort levels but then leaves firms free to compete in prices. The planning problem is to maximise welfare net of effort costs:

$$\max_{e_L,e_H} W = \underbrace{n\left[qD_H^{\alpha} + (1-q)D_L^{\alpha}\right]}_{\text{aggregate utility}} - \underbrace{\frac{\alpha w}{n}}_{\text{aggregate production cost}} - \underbrace{n\left[qbe_H^2 + (1-q)be_L^2\right]}_{\text{aggregate restructuring cost}}.$$

The first-order conditions are:

$$-q\alpha D_H^{\alpha-1}\frac{\mathrm{d}D_H}{\mathrm{d}c_H} - (1-q)\alpha D_L^{\alpha-1}\frac{\mathrm{d}D_L}{\mathrm{d}c_H} = bqe_H^* \tag{A2.10}$$

$$-q\alpha D_H^{\alpha-1}\frac{\mathrm{d}D_H}{\mathrm{d}c_L} - (1-q)\alpha D_L^{\alpha-1}\frac{\mathrm{d}D_L}{\mathrm{d}c_L} = bqe_L^*. \tag{A2.11}$$

To illustrate one interesting outcome, consider the case where $q = \frac{1}{2}$. Using the fact that $D_H^{\alpha-1}c_L = D_L^{\alpha-1}c_H$, the first-order conditions become

$$-\alpha D_H^{\alpha-1}\left(\frac{\mathrm{d}D_H}{\mathrm{d}c_H}+\frac{c_L}{c_H}\frac{\mathrm{d}D_L}{\mathrm{d}c_H}\right)=be_H^* \tag{A2.12}$$

$$-\alpha D_H^{\alpha-1}\left(\frac{\mathrm{d}D_H}{\mathrm{d}c_L}+\frac{c_L}{c_H}\frac{\mathrm{d}D_L}{\mathrm{d}c_L}\right)=be_L^*. \tag{A2.13}$$

Now consider what happens as the degree of cost asymmetry grows large. If $c_H/c_L \to \infty$, the socially efficient e_H* remains positive but e_L* goes to the zero. To see this, note that $\mathrm{d}D_H/\mathrm{d}c_H < 0, < \mathrm{d}D_H/\mathrm{d}c_L$, so that as $c_H/c_L \to \infty$, the LHS of (A2.12) remains positive and bounded away from zero, whereas the LHS of (A2.13) becomes negative, thereby delivering the corner solution $e_L^* = 0$. In this case the social planner would allocate effort toward high-cost firms in order to restore full symmetry whereas, as we have argued above, under laissez-faire it is the low-cost firms that invests more in cost reduction. More generally, as in the circular model, we cannot draw definitive conclusions on the comparison between laissez-faire and efficient restructuring without imposing further structure on the model.

Entry

Suppose that the number of incumbent firms in the market is sufficiently large that strategic considerations can be ignored when analysing individual entry decisions. By investing effort cost $C(P_i) = \gamma P_i^2$, a potential entrant succeeds in entering the market with probability P_i. A potential entrant with unit production cost c_i will choose its innovative (or R&D) effort P_i to

$$\max_{P_i} P_i \Pi_i - \gamma P_i^2,$$

where, for $i \in \{H, L\}$,

$$\Pi_i = \left(\frac{1}{\alpha}-1\right)c_i\left(\frac{c_i}{\alpha}\right)^{\frac{1}{\alpha-1}}K_{n',q'}$$

is the post-entry equilibrium profit of a firm with unit production cost c_i, n' is the (equilibrium) post-entry number of firms; q' is the (equilibrium) post-entry fraction of high-cost firms[2] (both n' and q' are rationally anticipated by each potential entrant), and

$$K_{n',q'} = \left\{\frac{n'\left[q'D_H^\alpha + (1-q')D_L^\alpha\right]}{w}\right\}^{\frac{1}{\alpha-1}}.$$

The first-order conditions imply equilibrium probabilities of entry

$$P_i = \frac{\Pi_i}{2\gamma}$$

where, using (A2.5), (A2.6) and the expressions for $A(\alpha)$ and $B(\alpha)$, we have:

[2] As in Section 4 in the text, we have:

$$n' = n + \theta N P_H + (1-\theta) N P_L$$
$$q' = \frac{nq + \theta N P_H}{n + \theta N P_H + (1-\theta) N P_L}$$

where P_i is the equilibrium probability of entry for a potential entrant with cost c_i, N is the number of potential entrants and is the fraction of high-cost firms among them.

$$\Pi_L = \left(\frac{c_H}{c_L}\right)^{\frac{\alpha}{1-\alpha}} \left[q' + (1-q')\left(\frac{c_H}{c_L}\right)^{\frac{\alpha}{1-\alpha}}\right] Z(\alpha) \tag{A2.14}$$

and

$$\Pi_H = \left[q' + (1-q')\left(\frac{c_H}{c_L}\right)^{\frac{\alpha}{1-\alpha}}\right] Z(\alpha) \tag{A2.15}$$

with

$$Z(\alpha) = (\bar{c})^{\frac{\alpha}{\alpha-1}} D_S^{\frac{\alpha}{\alpha-1}} \alpha^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha}\right).$$

Since $c_H > c_L$, the ratio P_L/P_H must increase with α, and the more so the greater the degree of cost asymmetry as measured by c_H/c_L. However, unlike in the circular model developed in the text (Proposition 6), we cannot show analytically that for c_H/c_L sufficiently large, the probability P_L should end up actually increasing with the degree of product market competition (measured by α), nor can we say much about the behaviour of P_L in relation to the Herfindahl index. The reason is that we cannot sign the derivative $\mathrm{d}Z/\mathrm{d}\alpha$.

Appendix 3

This Appendix shows that the selection effect of increasing competition holds in a Cournot model with horizontal product differentiation and asymmetric costs. For brevity we focus on identifying the selection effect on the relative levels of output by high- and low-cost firms in equilibrium.

There are n goods, each produced by a single firm. A proportion q of firms have unit cost c_H and the remaining firms have c_L where $c_H > c_L$. We analyse Nash equilibria in quantities. Taking the total number of firms to be large, and given that the other firms with unit cost c_H (c_L) set output level x_H (x_L), each firm i solves

$$\max_{x_i} (p_i - c_i) x_i$$

where $p_i(x_i, x_H, x_L, q)$ is the inverse demand curve faced by firm i. We assume the linear demand specification:

$$p_i = a - bx_i - \sigma x_{-i}$$

where $a > c_H$ and $x_{-i} = \sum_j x_j$. To simplify algebra, we analyse the case of arge n, so that $x_{-i} = n[qx_H + (1-q)x_L]$. The parameter σ captures the degree of competition (degree of strategic substitutability). A rise in σ corresponds to greater competition. From first order conditions, we get the following equilibrium output levels:

$$x_L^* = \frac{a - c_L + n\sigma q \dfrac{\Delta c}{2b}}{2b + n\sigma}$$

$$x_H^* = \frac{a - c_H - n\sigma(1-q)\dfrac{\Delta c}{2b}}{2b + n\sigma}.$$

The selection effect on output can be seen directly from looking at the ratio $x_L^*/x_H^* = 2b(a - c_L) + n\sigma q\Delta c/2b(a - c_H) - n\sigma(1-q)\Delta c$, which rises unambiguously when σ increases. An increase in σ reduces the output level for high cost firms unambiguously. However, the output of low-cost firms can either rise or fall, depending on the degree of cost

asymmetry and the proportion of high-cost firms. When Δc or q is large enough, an intensification of competition raises the output level of low cost firms because the selection effect is strong.

Also note that since $x_L^* > x_H^*$, low-cost firms have a greater incentive than high-cost firms to undertake cost reduction (market share effect) in the Cournot model, as in the circular and Dixit-Stiglitz models. The selection effect – the fact that x_L^*/x_H^* increases in σ – also gives low cost potential entrants a greater incentive to enter when competition is more intense but, as in the other models, this must be weighed against the reduction in the price-cost margin.