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Technical Appendix to

INTERNATIONAL RESERVE HOLDINGS WITH SOVEREIGN RISK AND COSTLY TAX COLLECTION

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A. The behaviour of a soft regime

A higher probability of a soft future administration leads to higher borrowing and lower international reserves accumulation in period 1:

$$\frac{\mathrm{d}B}{\mathrm{d}\phi} < 0;\ \frac{\mathrm{d}R}{\mathrm{d}\phi} > 0. \tag{A1}$$

We show this will be the case in an internal equilibrium where the complementarity between reserves and borrowing is not too great.

Note that the first-order conditions for determining the behaviour of the tough policy maker in the first period, (28) and (29), are:

$$\frac{\partial V_{|\mathrm{Tough 1st period}}}{\partial B} = 0;\ \frac{\partial V_{|\mathrm{Tough 1st period}}}{\partial R} = 0. \tag{A2}$$

Hence,

$$\begin{pmatrix} \dfrac{\partial^2 V_{|\mathrm{T}}}{\partial B^2} & \dfrac{\partial^2 V_{|\mathrm{T}}}{\partial R\, \partial B} \\ \dfrac{\partial^2_{|\mathrm{T}}}{\partial B \partial R} & \dfrac{\partial^2 V_{|\mathrm{T}}}{\partial R^2} \end{pmatrix} \begin{pmatrix} \dfrac{\mathrm{d}B}{\mathrm{d}\phi} \\ \dfrac{\mathrm{d}R}{\mathrm{d}\phi} \end{pmatrix} = \frac{1+r_f}{1+\rho} \begin{Bmatrix} \int_{\varepsilon^*}^{\bar{\delta}} u'(c_2)[1+\Gamma'(\xi_2)]f(\varepsilon)\mathrm{d}\varepsilon \\ -\int_{-\bar{\delta}}^{\bar{\delta}} u'(c_2)[1+\Gamma'(\xi_2)]f(\varepsilon)\mathrm{d}\varepsilon \end{Bmatrix}, \tag{A3}$$

where $V_{|\mathrm{T}}$ is a shortened notation for $V_{|\mathrm{Tough\ 1st\ period}}$.

Thus,

$$\begin{aligned} \frac{\mathrm{d}B}{\mathrm{d}\phi} &= \frac{1+r_f}{1+\rho} \left\{ \int_{\varepsilon^*}^{\bar{\delta}} u'(c_2)[1+\Gamma'(\xi_2)]f(\varepsilon)\mathrm{d}\varepsilon \frac{\partial^2 V_{|\mathrm{T}}}{\partial R^2} \right. \\ &\quad \left. + \int_{-\bar{\delta}}^{\bar{\delta}} u'(c_2)[1+\Gamma'(\xi_2)]f(\varepsilon)\mathrm{d}\varepsilon \frac{\partial^2 V_{|\mathrm{T}}}{\partial R \partial B} \right\} \Big/ D \\ \frac{\mathrm{d}R}{\mathrm{d}\phi} &= \frac{1+r_f}{1+\rho} \left\{ -\int_{-\bar{\delta}}^{\bar{\delta}} u'(c_2)[1+\Gamma'(\xi_2)]f(\varepsilon)\mathrm{d}\varepsilon \frac{\partial^2 V_{|\mathrm{T}}}{\partial R^2} \right. \\ &\quad \left. - \int_{\varepsilon^*}^{\bar{\delta}} u'(c_2)[1+\Gamma'(\xi_2)]f(\varepsilon)\mathrm{d}\varepsilon \frac{\partial^2 V_{|\mathrm{T}}}{\partial B \partial R} \right\} \Big/ D \end{aligned} \tag{A4}$$

$$\text{where } D = \begin{vmatrix} \dfrac{\partial^2 V_{|\mathrm{T}}}{\partial B^2} & \dfrac{\partial^2 V_{|\mathrm{T}}}{\partial R\,\partial B} \\ \dfrac{\partial^2 V_{|\mathrm{T}}}{\partial B \partial R} & \dfrac{\partial^2 V_{|\mathrm{T}}}{\partial R^2} \end{vmatrix}.$$

Applying (27) we infer that $\dfrac{\partial^2 V_{|\mathrm{T}}}{\partial B^2} < 0$; $\dfrac{\partial^2 V_{|\mathrm{T}}}{\partial R^2} < 0$; $\dfrac{\partial^2 V_{|\mathrm{T}}}{\partial B \partial R} > 0$. The second order condition for internal optimisation implies that D is positive. This will be the case if the complementarity between B and R (as measured by $\partial^2 V_{|\mathrm{T}}/\partial B \partial R$) is smaller that the geometric average of the direct effects of B and R (i.e., if $\sqrt{\dfrac{\partial^2 V_{|\mathrm{T}}}{\partial B^2}\dfrac{\partial^2 V_{|\mathrm{T}}}{\partial R^2}} > \dfrac{\partial^2 V_{|\mathrm{T}}}{\partial B \partial R}$). Inspection of (A4) reveals that a sufficient condition to sign the impact of changing the probability of a future soft policy maker is that the complementarity between B and R is low enough. In these circumstances.[1]

$$\frac{\mathrm{d}B}{\mathrm{d}\phi} < 0; \ \frac{\mathrm{d}R}{\mathrm{d}\phi} > 0. \tag{A5}$$

B. Investment and political uncertainty

We extend the model in the text to allow for private investment. This extension allows us to derive expected second-period output endogenously. In addition, the supply of external borrowing facing the economy in the first period is endogenously determined by policy uncertainty.

Suppose that first-period investment is financed out of the private sector disposable income. Hence, second-period output depends positively on first-period investment, I:

$$Y_2 = g(I)(1+\varepsilon); \text{ where } g(0) = 1; \ g' > 0; \ g'' < 0. \tag{B1}$$

We preserve all the other assumptions of the model. The representative agent's expected utility in the presence of policy uncertainty is:

$$u(1 - t_1 - I) + \frac{\phi}{1+\rho}\int_{-\bar{\delta}}^{\bar{\delta}} u(g(I)(1+\varepsilon)\{1 - t_2[\xi_2(\varepsilon)]\})f(\varepsilon)\mathrm{d}\varepsilon$$
$$+ \frac{1-\phi}{1+\rho}\int_{-\bar{\delta}}^{\bar{\delta}} u[g(I)(1+\varepsilon)(1-t_m)]f(\varepsilon)\mathrm{d}\varepsilon \tag{B2}$$

where $t_2[\xi_2(\varepsilon)]$ denotes the state-dependent second-period tax rate under the tough fiscal regime; see (9). The optimal investment is determined by maximising expected utility, resulting in the following FOC:[2]

[1] It is easy to confirm that for a given B, a higher return on R would increase the optimal demand for R. Similarly, for a given R, higher expected borrowing costs would reduce B. Due to the complementarity between B and R, there are secondary effects: the increase in R triggered by higher returns will increase B, whereas the drop in B induced by the higher cost of borrowing will reduce R. The direct effects will dominate the secondary effects only if the complementarity between B and R is not too great.

[2] We assume a competitive equilibrium, where each entrepreneur's investment decision is too small to affect tax rates. Hence, the entrepreneur treats $t_2[\xi_2(\varepsilon)]$ as exogenous.

$$u_1'(1-t_1-I)=\frac{g'(I)}{1+\rho}\left[\begin{array}{c}\phi\int_{-\bar{\delta}}^{\bar{\delta}} u_2'(g(I)(1+\varepsilon)\{1-t_2[\xi_2(\varepsilon)]\})(1+\varepsilon)\{1-t_2[\xi_2(\varepsilon)]\}f(\varepsilon)\mathrm{d}\varepsilon\\ +(1-\phi)\int_{-\bar{\delta}}^{\bar{\delta}} u_2'[g(I)(1+\varepsilon)(1-t_m)](1+\varepsilon)(1-t_m)f(\varepsilon)\mathrm{d}\varepsilon\end{array}\right]. \quad \text{(B3)}$$

Optimal investment equates the marginal cost of private funds (first-period marginal utility) to the expected marginal benefit of investment [the RHS of (B3)]. The marginal benefit of investment equals the product of the discounted second-period marginal utility and the marginal product of capital net of taxes.

Applying (B3), it follows that a lower probability of a soft fiscal regime in the second period (ø > 0) increases first-period investment if it increases the expected marginal utility of the marginal product of capital, the RHS of (B3). Applying the implicit function theorem it follows that:

$$\mathrm{sgn}\left(\frac{\mathrm{d}I}{\mathrm{d}\phi}\right)=\mathrm{sgn}\left\{\int_{-\bar{\delta}}^{\bar{\delta}}\left[\begin{array}{c}u_2'(g(I)(1+\varepsilon)\{1-t_2[\xi_2(\varepsilon)]\})\{1-t_2[\xi_2(\varepsilon)]\}\\ -u_2'[g(I)(1+\varepsilon)(1-t_m)](1-t_m)\end{array}\right](1+\varepsilon)f(\varepsilon)\mathrm{d}\varepsilon\right\}. \quad \text{(B4)}$$

Recall that the second-period tax rate is lower under the tough policy maker, so $t_2[\xi_2(\varepsilon)] \leq t_m$. If agents are risk neutral, $\mathrm{sgn}(\mathrm{d}I/\mathrm{d}\phi) > 0$. All that matters is the impact of the higher probability of a tough policy maker on the expected future tax rate. The lower probability of a soft fiscal regime implies a lower expected future tax rate and hence a higher expected marginal product of capital, encouraging more present investment.

In general, however, the sign of (B4) is determined by the degree of risk aversion: it is positive (negative) when the coefficient of relative risk aversion is below (above) unity.[3] Thus greater uncertainty about the future regime and future returns has an ambiguous effect on current investment. The outcome depends on the shape of the utility function.

[3] To illustrate, suppose that the utility is given by $u(c) = c^{1-v}/(1-v)$, where v is the relative risk aversion, $v \geq 0$. It is easy to verify that in this case,

$$\mathrm{sgn}\left(\frac{\mathrm{d}I}{\mathrm{d}\phi}\right)=\mathrm{sgn}\left\{\int_{-\bar{\delta}}^{\bar{\delta}}\left[\begin{array}{c}(g(I)(1+\varepsilon)\{1-t_2[\xi_2(\varepsilon)]\})^{1-v}\\ -[g(I)(1+\varepsilon)(1-t_m)]^{1-v}\end{array}\right]f(\varepsilon)\mathrm{d}\varepsilon\right\}=\mathrm{sgn}(1-v).$$