The Economic Journal, **114** (*October*), © Royal Economic Society 2004. Published by Blackwell Publishing, 9600 Garsington Road, Oxford OX4 2DQ, UK and 350 Main Street, Malden, MA 02148, USA.

Technical Appendix to

GOING ALONE TOGETHER: JOINT OUTSIDE OPTIONS IN BILATERAL NEGOTIATIONS

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ECONOMIC JOURNAL, vol. 114 (*October*), pp. 943–960

Appendix

All subgames of the same kind starting with a move by player i are identical and have the same set of equilibrium payoffs. Then, let $\overline{g}_j^i$ and $\underline{g}_j^i$ (with i, $j = 1$, 2) be the supremum and the infimum payoffs, respectively, for player j in any subgame perfect equilibrium of subgames of type G^i where player i makes an offer. Similarly, let $\overline{d}_j^i$ and $\underline{d}_j^i$, respectively, be the supremum and infimum equilibrium payoff to player j in any subgame perfect equilibrium of subgames of type D^i where player i has to decide whether to accept taking the JOO, or to decline it and make a counteroffer in the next round.

Subgame perfection requires that the following hold:

$$\overline{g}_i^i \leq f_i(\max\left[p\underline{g}_j^j, \underline{d}_j^i\right]), \underline{g}_i^i \geq f_i(\max\left[p\overline{g}_i^i, \overline{d}_j^i\right]) \quad i,j = 1,2 \tag{2}$$

$$\overline{d}_i^i \leq max\left[m_i, p\overline{g}_i^i\right], \underline{d}_i^i \geq max\left[m_i, p\underline{g}_i^i\right] \quad i,j = 1,2 \tag{3}$$

Proof of Proposition 1. Denote

$$g^i = (g_i^i, g_j^i) = (f_i(m_j), m_j).$$

The following strategies for player $i = 1$, 2 support (g_1^1, g_2^1) as an s.p.e. negotiated alternative:

(*i*) propose the alternative g^i;
(*ii*) reject any alternative which yields him any $x < g_i^j$ and accept any $x \geq g_i^j$;
(*iii*) always accept the JOO when player j proposes it;
(*iv*) always propose to take the JOO when rejecting an offer.

Checking for subgame perfection is straightforward. Notice that the condition in the statement $m_i \geq pf_i(m_j) \forall i, j$ ensures optimality of (*iii*) and (*iv*). For (*iii*), by parts (*i*) and (*ii*) of the equilibrium strategies it must be the case that in subgames of type D^i player i prefers the JOO to the expected continuation payoff g_i^i. Similarly, for (*iv*) to be optimal at r, it must be the case that the payoff that player i gets when rejecting an offer in G^j and proposing to take the JOO is not lower than the expected payoff from inducing G^i in $r + 1$ instead. Given part (*iii*) of j's equilibrium strategy, the former payoff is m_i. Therefore this leads to the same inequality considered for (*iii*). ■

Proof of Proposition 2. The following lemma gives conditions under which the maximum s.p.e. payoff for a player in subgames where he is the proposer is greater than the maximum s.p.e. payoff to that player in subgames when the opponent is a proposer, and will be needed in the proof of Proposition 2.

LEMMA 8 *If, for some i,* $m_i \leq p\overline{g}_i^i$, *then* $\overline{g}_i^i \geq \overline{g}_i^j\ \forall p \in (0, 1)$.

Proof. Suppose, by contradiction, that

$$\overline{g}_i^i < \overline{g}_i^j.$$

We will show that in this case i would be willing to accept $\overline{g}_i^j - \eta$ in the subgame G^j for some small η, thus contradicting the definition of $\overline{g}_i^j$.

In fact, suppose that i rejected the outcome yielding him $\overline{g}_i^j - \eta$ and made a counteroffer. Then he would get at most $p\overline{g}_i^i$: but

$$\begin{aligned} p\overline{g}_i^i &< \overline{g}_i^i < \overline{g}_i^j \\ &\Rightarrow p\overline{g}_i^i < \overline{g}_i^j - \eta \end{aligned}$$

for η sufficiently small. Suppose, on the other hand, that i rejected the outcome yielding him $\overline{g}_i^j - \eta$ and proposed the JOO. Then, if j accepted, i could obtain at most

$$m_i \leq p\overline{g}_i^i < p\overline{g}_i^j < \overline{g}_i^j - \eta$$

for a small η. If j rejected the JOO, i could obtain at most

$$p\overline{g}_i^j < \overline{g}_i^j - \eta$$

for a small η. ■

Proof of Proposition 2. We will show that

$$\overline{g}_i^i \leq f_i(m_j) \leq \underline{g}_i^i \text{ with } i, j = 1, 2,$$

from which the conclusion of the statement follows. We proceed in two steps:

Step 1: $\overline{g}_i^i \leq f_i(m_j)$.
Suppose to the contrary that $\overline{g}_i^i > f_i(m_j)$. Distinguish two cases:

1. $m_i > p\overline{g}_i^i$
2. $m_i \leq p\overline{g}_i^i$

We will show that, at an equilibrium, in both cases there is a move by player j which is more profitable than accepting $[\overline{g}_i^i, f_j(\overline{g}_i^i)]$.

Case 1. Consider the following action by player j: reject the alternative $[\overline{g}_i^i, f_j(\overline{g}_i^i)]$ and propose the JOO. Player i would accept this proposal in equilibrium, since by rejecting he can get at most $p\overline{g}_i^i$, while by accepting he gets $m_i > p\overline{g}_i^i$ by assumption. Then by this action player j gets

$$m_j > f_j(\overline{g}_i^i)$$

since $\overline{g}_i^i > f_i(m_j)$ (by assumption) and $f_j(\cdot)$ is monotonically decreasing, hence

$$f_j(\overline{g}_i^i) < f_j\ [f_i(m_j)] = m_j$$

Case 2. Player j can profitably reject and propose the alternative $\{p(\overline{g}_i^i+\eta), f_j[p(\overline{g}_i^i+\eta)]\}$. In equilibrium player i accepts this proposal, since otherwise he could either (i) make a counteroffer, obtaining at most

$$p\overline{g}_i^i < p(\overline{g}_i^i+\eta)$$

or (ii) propose to take the JOO; then, if player j concedes on taking the JOO, player i gets

$$m_i \leq p\overline{g}_i^i < p(\overline{g}_i^i+\eta)$$

(the first inequality following from the definition of Case 2). If the JOO is rejected, player i gets at most

$$p\overline{g}_i^j \leq p\overline{g}_i^i < p(\overline{g}_i^i+\eta)$$

(where the first inequality follows from lemma 8).

Finally, player j profits from this counterproposal iff:

$$pf_j[p(\overline{g}_i^i+\eta)] > f_j(\overline{g}_i^i)$$

that is - given that $f_i(\cdot)$ is monotonically decreasing - iff:

$$f_i\{pf_j[p(\overline{g}_i^i+\eta)]\} < f_i[f_j(\overline{g}_i^i)] = \overline{g}_i^i \qquad (*)$$

But by balancedness and the condition defining case 2:

$$p\overline{g}_i^i \geq m_i > p\rho_i^i = pf_i[pf_j(p\rho_i^i)] \Rightarrow \overline{g}_i^i > f_i[pf_j(p\rho_i^i)] = \rho_i^i.$$

This implies ($*$) for η small, because ρ_i^i is the unique fixed point of $F(x) = f_i[pf_j(px)]$, $\rho_i^i > 0$; thus, we must have $x > F(x)\,\forall x > \rho_i^i$.

Step 2: $f_i(m_j) \leq \underline{g}_i^i$.

We start showing that at an equilibrium player j cannot improve on m_j. To see this, suppose first that player j rejected the proposal $(f_i(m_j), m_j)$ and made a counter-offer; then, by Step 1 the maximum he could get by this action is $pf_j(m_i)$. Thus, rejecting pays only if $pf_j(m_i) > m_j$. But this is a contradiction given that m is balanced. Hence, player j would not find it profitable to follow this action.

On the other hand, suppose that player j proposed to take up the JOO after rejecting $(f_i(m_j), m_j)$. In an s.p.e., player i would accept, since by Step 1 and the above argument he would obtain at most $pf_i(m_j) < m_i$ by rejecting and inducing a game G^i.

This shows that, if the JOO is balanced, at an s.p.e. player j will accept any proposal yielding him strictly more than m_j, and will be indifferent between (a) accepting an offer m_j and (b) rejecting and proposing the JOO. However, if $f_i(m_j) > m_i$ there cannot be an s.p.e. where player j follows action (b) after player i's proposing the alternative $(f_i(m_j), m_j)$, since then player i could improve on his payoff by proposing $(f_i(m_j+\eta), m_j+\eta)$ instead, which, for small η, would yield him a payoff greater than m_i he would obtain in the JOO,[1] and which would be accepted.

Proof of Proposition 4. For the 'if' part, we describe strategies supporting the s.p.e.. For player $i = 1, 2$:

(i) propose ρ^i;
(ii) accept only alternatives which yield at least $p\rho_i^i$;
(iii) do not propose to take the JOO when rejecting;
(iv) reject the JOO when $m_i \leq p\rho_i^i$, and do not reject it otherwise.

[1] Note that $f_i(m_j) \geq m_i$, with equality holding if m is efficient.

The optimality of these strategies is checked easily. We limit ourselves to note that if m is not balanced, then there exists i such that $m_i \le p\rho_i^i$, which ensures the optimality of (*iii*).

For the 'only if' part, recall that when the JOO is balanced the only s.p.e. payoff is as described in Proposition 2. ■

Proof of Proposition 5. We first need to prove a result similar to Lemma 8:

LEMMA 9 *If* $m_i \le p\rho_i^i$ *and* $\overline{g}_i^i > \rho_i^i$ *for some i, then* $\overline{g}_i^i > \overline{g}_i^j \ \forall p \in (0,1)$.

Proof. Suppose, by contradiction, that

$$\overline{g}_i^i \le \overline{g}_i^j.$$

Then also

$$\overline{g}_i^j > \rho_i^i.$$

We will show that in this case i would be willing to accept $\overline{g}_i^j - \eta$ in the subgame G^j for some small η, thus contradicting the definition of $\overline{g}_i^j$.

In fact, suppose that i rejected the alternative yielding him $\overline{g}_i^j - \eta$ and made a counteroffer. Then he would get at most $p\overline{g}_i^i$: but

$$p\overline{g}_i^i < \overline{g}_i^i \le \overline{g}_i^j \Rightarrow p\overline{g}_i^i < \overline{g}_i^j - \eta$$

for η sufficiently small. Suppose, on the other hand, that i rejected the alternative yielding him $\overline{g}_i^j - \eta$ and proposed to take the JOO. Then, if j accepted, i could obtain at most

$$m_i \le p\rho_i^i < \rho_i^i < \overline{g}_i^j - \eta$$

for a small η. If j declined the JOO, i could obtain at most

$$p\overline{g}_i^j < \overline{g}_i^j - \eta$$

for a small η. ■

Proof of Proposition 5 Assuming that p satisfies the condition in the statement, we deal with three intermediary steps.

Step 1: $m_i \le p\rho_i^i \Rightarrow \overline{g}_i^i \le \rho_i^i$.

Suppose to the contrary that

$$\overline{g}_i^i > \rho_i^i \Leftrightarrow f_j(\overline{g}_i^i) < f_j(\rho_i^i) = \rho_j^i.$$

We will show that in an s.p.e. where $\overline{g}_i^i$ satisfies this condition a contradiction results, because j could reject any proposal yielding him less than ρ_j^i and offer to player i the share $p(\overline{g}_i^i + \eta)$, which would be accepted at an s.p.e. Then j would be better off than by accepting $f_j(\overline{g}_i^i)$ because for η small:

$$\overline{g}_i^i > \rho_i^i \Rightarrow \overline{g}_i^i > p(\rho_i^i + \eta) \Leftrightarrow f_j(\overline{g}_i^i) < f_j[p(\rho_i^i + \eta)].$$

To see that i would accept the proposal at an s.p.e., observe that if he made a counteroffer he could get at most

$$p\overline{g}_i^i < p(\overline{g}_i^i + \eta)$$

On the other hand, if he proposed to take the JOO, either j would accept and i would get

$$m_i < p(\overline{g}_i^i + \eta)$$

by the assumption $m_i \leq p\rho_i^i < p\overline{g}_i^i$; or j would decline and i would get at most

$$p\overline{g}_i^j < p(\overline{g}_i^i + \eta)$$

by Lemma 9.

Step 2: $m_i \leq p\rho_i^i,\ m_j > p\rho_j^j \Rightarrow p_m = m_i/f_i(m_j)$.

Note first that $p_m = m_i/f_i(m_j) \Leftrightarrow m_i f_j(m_i) < m_j f_i(m_j)$. To see that this inequality is implied by the assumptions, we give a simple geometric argument. Consider the point $x = (x_i, x_j) = (p\rho_i^i, p\rho_j^j) \in S$. Because ρ^i and ρ^j lie on the same hyperbola, x is such that $x_i f_i(x_j) = x_j f_j(x_i)$. Thus any point $y \in S$ with $y_i < x_i$ and $y_j > x_j$ is such that $y_i f_i(y_j) < x_i f_i(x_j)$, while $y_j f_j(y_i) > x_j f_j(x_i)$ (see Figure 5, which depicts the case for $y_1 < x_1$ and $y_2 > x_2$). Therefore the conditions $m_i \leq p\rho_i^i$, $m_j > p\rho_j^j$ imply that $m_i f_i(m_j) < m_j f_j(m_i)$.

Step 3: $m_i \leq p\rho_i^i,\ m_j > p\rho_j^j \Rightarrow \underline{g}_i^i \geq \rho_i^i$.

Suppose to the contrary that

$$\underline{g}_i^i < \rho_i^i.$$

Then there exists an s.p.e. of some subgame in which j would reject any alternative yielding him ρ_j^i. Thus, suppose it was optimal for j to reject and induce a subgame G^j. By so doing he would get at most

$$f_j[\max(p\underline{g}_i^i, \underline{d}_i^i)] \leq f_j\left(p\underline{g}_i^i\right)$$

in the next round, yielding the contradiction

$$\underline{g}_i^i \geq f_i\left[pf_j\left(p\underline{g}_i^i\right)\right] \Leftrightarrow \underline{g}_i^i \geq \rho_i^i$$

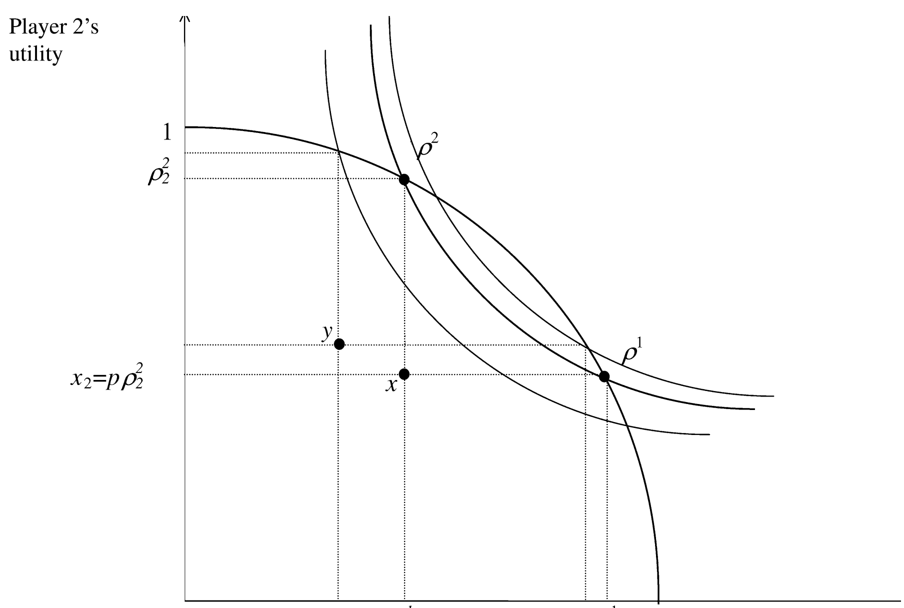

Fig. 5. *Step 2 of the Proof of Proposition 4*

where the last equivalence follows from the argument in Step 1 of Case 2 of Proposition 2.

Suppose then that it was optimal for j to reject and propose to take the JOO. If i declined, then j would get at most $pf_j\left(\underline{g}_i^i\right)$, so that

$$\underline{g}_i^i \geq f_i\left[pf_j\left(\underline{g}_i^i\right)\right] = F\left(\frac{\underline{g}_i^i}{p}\right) > F\left(\underline{g}_i^i\right)$$

where recall that $F(x) \equiv f_i[pf_j(px)]$.

But this is a contradiction by a now familiar argument. Therefore, if it was optimal for j to reject at an s.p.e., i would concede to take the JOO. This can only be the case if

$$m_i \geq p\underline{g}_i^i \Leftrightarrow \frac{m_i}{p} \geq \underline{g}_i^i.$$

Thus, when rejecting optimally j gets exactly m_j, so that

$$\underline{g}_i^i \geq f_i(m_j) \geq m_i.$$

The last two inequalities are compatible only if

$$\frac{m_i}{p} \geq f_i(m_j) \Leftrightarrow p \leq \frac{m_i}{f_i(m_j)}$$

If it is the case that $p_m = m_i/f_i(m_j)$, then the above contradicts the assumption that $p > p_m$. But by step 2 it must indeed be $p_m = m_i/f_i(m_j)$. This concludes the proof of step 3.

To conclude the proof, observe that only two cases can hold:

$$(a)\ \ m_i \leq p\rho_i^i, m_j \leq p\rho_j^j$$

$$(b)\ \ m_i \leq p\rho_i^i, m_j > p\rho_j^j.$$

In case (a), step 1 yields

$$\bar{g}_i^i \leq \rho_i^i \text{ and } \bar{g}_j^j \leq \rho_j^j.$$

In case (b), step 3 yields

$$\bar{g}_i^i \leq \rho_i^i \leq \underline{g}_i^i.$$

In both cases, it follows from standard reasoning that $\rho_i^i = \underline{g}_i^i = \bar{g}_i^i$ and $\rho_j^j = \bar{g}_j^j = \underline{g}_j^j$. ■

Proof of Proposition 6. To see that the intervals described in the statement are not empty, recall that $m_j < p\rho_j^j \Rightarrow f_i(m_j) > f_i\left(p\rho_j^j\right) = \rho_i^i$, where the equality derives from the definition $f_j\left(\rho_i^i\right) = p\rho_j^j$, $i, j = 1, 2$, by applying f_i on both sides. Furthermore, since $m_i > pf_i(m_j)$, the fact just proved that $f_i(m_j) > \rho_i^i$ implies $m_i > p\rho_i^i$.

The rest of the proof is divided into two parts. In the first part we show that no alternative which yields player i a payoff outside the ranges specified in the proposition can be supported in equilibrium. In part II we describe a pair of strategies which support the s.p.e. outcome introduced above.

Part I: Payoff bounds

Step 1: $\bar{g}_i^i \leq f_i(m_j)$.

The proof of this step is identical to the proof of Step 1 in Proposition 2. Note that the conditions in the statements of both the present proposition and Proposition 2 are the same, except that in the present case we are not assuming balance. However, we have proved above that the inequality $m_i > p\rho_i^i$ holds, and it is easy to check that it is only this part of the balance condition which is used in the proof of Step 1 of Proposition 2.

Step 2: $\underline{g}_i^i \geq \rho_i^i$.

We show that player j would receive a payoff of at most $\rho_j^i = p\rho_j^j$ when responding in G^i. When rejecting, player j can either (i) propose to take the JOO, or (ii) make a counteroffer. If (i), player i would accept, since in this way his payoff exceeds the highest payoff he could otherwise obtain by declining the JOO to make a counteroffer, given that

$$m_i > pf_i(m_j) \geq p\overline{g}_i^i$$

where the weak inequality follows from Step 1. Thus, j would obtain $m_j < p\rho_j^j$. Alternatively, if (ii), j would induce a subgame G^j. By so doing he would get at most

$$f_j\left[\max(p\underline{g}_i^i, \underline{d}_i^i)\right] \leq f_j\left(p\underline{g}_i^i\right)$$

in the next round, yielding

$$\underline{g}_i^i \geq f_i\left[pf_j\left(p\underline{g}_i^i\right)\right]$$
$$\Leftrightarrow \underline{g}_i^i \geq \rho_i^i$$

where the last equivalence follows from the argument given in Step 1 of Case 2 of Proposition 2. This shows that player j's payoff is not greater than $p\rho_j^j$, as desired.

Step 3: $\underline{g}_i^j \geq p\rho_i^i$.

Suppose to the contrary:

$$\underline{g}_i^j < p\rho_i^i \Rightarrow f_j\left(\underline{g}_i^j\right) > f_j\left(p\rho_i^i\right) \Leftrightarrow \overline{g}_j^j > \rho_j^j.$$

Then player i could reject and propose in the following round an alternative yielding player j a payoff of $p\left(\overline{g}_j^j + \eta\right) > p\rho_j^j$. Player j will accept, because by rejecting he can obtain either $p\overline{g}_j^j$, by making a counterproposal in the subsequent round; or $m_j < p\rho_j^j < p\left(\overline{g}_j^j + \eta\right)$ if he proposes to take the JOO (which is accepted by player i, since, as argued in the previous step, $m_i > pf_i(m_j) \geq p\overline{g}_i^i$). Such a deviation is profitable for player i, if

$$pf_i\left[p\left(\overline{g}_j^j + \eta\right)\right] > \underline{g}_i^j = f_i\left(\overline{g}_j^j\right)$$

or, applying f_j on both sides,

$$f_j\left\{pf_i\left[p\left(\overline{g}_j^j + \eta\right)\right]\right\} < \overline{g}_j^j$$

which is true, for η small enough, if $\overline{g}_j^j > \rho_j^j$, as implied by our contradiction assumption.

Step 4: $\overline{g}_i^j \leq m_i$.

Player i cannot improve by rejecting an alternative yielding him m_i. Rejecting and making a counteroffer yields not more than $p\overline{g}_i^i \leq pf_i(m_j) < m_i$. Rejecting and proposing to take the JOO does not improve on m_i either, since: if player j accepts the JOO, i gets m_i; if player j rejects the JOO, player i can obtain at most $p\overline{g}_i^j$ in the following round.

Part II: Equilibrium strategies.

Let $y* \in \left[f_j(m_i), \rho_j^j\right]$. The arguments are standard and we just sketch the strategies supporting the equilibrium. For player i (player j, respectively): to claim x^* (y^*) for himself, to accept any proposal which yields him at least $f_i(y^*)$ ($f_j(x^*)$) and to reject any other proposal. In case of a deviation, play reverts to the strategies which yield the worst payoff for the deviator, that is either those supporting the Rubinstein s.p.e. payoff as specified in Proposition 4; or those supporting the JOO-type equilibrium, as specified in Proposition 1.

To check that the strategies sketched above constitute an s.p.e., consider first subgames in which player i makes an offer. If he proposed any alternative yielding him a payoff $x \neq x^*$, given his strategy player j would reject, and both players would revert to the Rubinstein equilibrium play: player j would propose the alternative ρ^j, which player i would accept, obtaining a lower expected payoff than the lowest possible value of x^* (i.e. $p\rho_i^j = p^2\rho_i^i < \rho_i^i$). Player j cannot improve on his payoff by accepting an alternative which yields a payoff different from $f_j(x^*)$, as by rejecting a disequilibrium offer and switching to the Rubinstein play he can secure a payoff whose expected value is $p\rho_j^j = f_j(\rho_i^i) \geq f_j(x^*)$ (where the inequality follows by applying f_j to both sides of $x^* \geq \rho_i^i$). Consider now subgames where player j is the proposer, so that agreement is reached on the alternative $(f_i(y^*), y^*)$. Player j cannot improve on his payoff by making a different offer, as given his strategy, player i would reject and then revert to play the JOO-type equilibrium strategies: he would propose to take the JOO, which player j will accept, obtaining a payoff of $m_j \leq f_j(m_i) \leq y^*$. Finally, player i cannot improve on his payoff by accepting an alternative which yields a payoff different from $f_i(y^*)$, as by rejecting a disequilibrium offer and switching to the JOO-type play he can secure a payoff of $m_i > f_i(y^*)$. ■

Proof of Proposition 7. We first describe an equilibrium given a certain configuration of parameter values and then we show that this configuration is compatible with $[\rho_1^1/p^R, f_1(m_2/p^R)]$ being not empty. Specifically, suppose that $m_i \geq pf_i(m_j)$, $i, j = 1, 2$ and that $m_2 < p\rho_2^2$ (so that (m_1, m_2) is not balanced); then both the Rubinstein and the JOO-type equilibria can obtain.

To describe the supporting strategies clearly, we make them conditional on 'states'. In the initial Equilibrium state: at round r and until round R, player 1 proposes the partition $(f_1[p^{R-r}f_2(z^*)];p^{R-r}f_2(z^*))$ and accepts only offers yielding at least m_1, while player 2 proposes the partition $[p^{R-r}z^*;f_2(p^{R-r}z^*)]$ and accepts only offers yielding at least $p\rho_2^2$; both players never propose to take the JOO; at round R, player i offers an alternative yielding player 1 a payoff of z^*, which is accepted. Any deviation[2] immediately triggers a change to a Punishment state for the deviating player. In the punishment state play reverts to the strategies supporting the worst equilibrium outcome for the deviator[3] (i.e. either those supporting the Rubinstein s.p.e. alternative, as specified in Proposition 3, or those supporting the JOO-type equilibrium, as specified in Proposition 1, as explained in the proof of Proposition 5). We now show that the payoffs specified above can be supported in equilibrium.

1. *Deviations by Player 1.* Consider first a deviation by player 1 in the *first round* ($r = 0$); there can actually be two such deviations:

(*i*) *Deviant proposal of an alternative.* A deviant proposal of an alternative can be either one which is rejected or one which is accepted. Suppose that player 1's proposed partition gives his opponent a share $p\rho_2^2 - \eta$ with $\eta > 0$. Then play reverts to the Rubinstein strategies. Player 2 rejects and makes a counteroffer in the following

[2] Except proposals of partitions which would be accepted.

[3] With these strategies, player 1's continuation payoff if he rejected a deviant offer yielding $m_1 + \eta$ would be m_1 (equilibrium payoff in the JOO-type equilibrium, obtained immediately). Consequently, it is optimal for player 1 to accept if $\eta > 0$, whereas he cannot improve on his equilibrium payoff by accepting if $\eta \leq 0$. Similarly, player 2's continuation payoff if he rejected a deviant offer yielding $p\rho_2^2 + \eta$ would be $p\rho_2^2$ (equilibrium payoff in the Rubinstein equilibrium in the following round). Consequently, it is optimal for player 2 to accept if $\eta > 0$, whereas he cannot improve on his equilibrium payoff by accepting if $\eta \leq 0$.

round yielding player 1 a payoff of $\rho_1^2 = p\rho_1^1$. On the other hand, by conforming to his equilibrium strategy player 1 could have obtained a payoff of z^* at round R; consequently, player 1 cannot profit from a deviation in the first round as long as:

$$p^2\rho_1^1 \leq p^R z^* \Rightarrow z^* \geq \frac{\rho_1^1}{p^{R-2}}. \tag{4}$$

Similarly, in even rounds other than the first ($r > 0$), a deviant offer by player 1 will not be profitable as long as $z* \geq \rho_1^1/p^{R-(r+2)}$; however, $\rho_1^1/p^{R-2} > \rho_1^1/p^{R-(r+2)}$, so that condition 4 above actually encompasses all deviant offers in rounds other than the first. Finally, note that at any even round r player 1 can never profit by proposing an alternative yielding player 2 a payoff $f_2(z) > p\rho_2^2 \geq m_2$ (which player 2 would accept), since otherwise player 1 would be left with the residual payoff

$$z < \rho_1^1.$$

But

$$z^* \geq \frac{\rho_1^1}{p^R} \Leftrightarrow p^{R-r}z^* \geq \frac{\rho_1^1}{p^r}$$

so that player 1 cannot improve on his equilibrium payoff $p^{R-r}z^*$.

(*ii*) *Deviant response to proposals to take the JOO.* Suppose that, after rejecting player 1's offer, player 2 unexpectedly proposes to take the JOO, triggering his own punishment state. Then, it is optimal for player 1 to punish this deviation by accepting it taking of the JOO, since in this way he obtains immediately the payoff in his best equilibrium, m_1. Similarly for even rounds other than the first.

Consider a deviation by player 1 in the *second round* ($r = 1$). Obviously, he cannot increase his payoff by accepting[4] player 2's offer of $p^R z^*$. Moreover, player 1 cannot profitably propose to take the JOO after rejecting player 2's proposal of an alternative, since this would trigger player 1's own punishment state, in which player 2 would reject, and counteroffer in the following round $\rho_1^2 = p\rho_1^1$, which is also not greater than player 1's equilibrium payoff ($p^2\rho_1^1 < p^{R-1}z^*$). Similarly for other deviations by player 1 in odd periods other than the first. Thus condition 3 imposes a lower bound on z^* which prevents profitable deviations by player 1.

2. *Deviations by Player 2.* Let us first consider a deviation by player 2 in the *first round.* Similarly to what we discussed for player 1, player 2 cannot increase his payoff by accepting player 1's equilibrium offer of $p^R f_2(z^*)$ (obvious). Next, suppose player 2 proposes to take the JOO after rejecting his opponent's proposal of an alternative. This triggers player 2's own punishment state, where player 1 accepts taking the JOO, yielding player 2 a payoff m_2. This is not greater than player 2's equilibrium payoff as long as

$$m_2 \leq p^R f_2(z^*) \Leftrightarrow z^* \leq f_1\left(\frac{m_2}{p^R}\right). \tag{5}$$

Similarly for other deviations by player 2 in even rounds r other than the first, which require for z^* a weaker condition than condition 5 [$z^* \leq f_1(m_2/p^{R-r})$].

Consider now a deviation by player 2 in the *second round* ($r = 1$). Symmetrically to the case of player 1, there can be two types of deviations:

[4] See also footnote 23.

(i') *Deviant proposal of an alternative.* Once again, a deviant proposal of an alternative can be either one which is rejected or one which is accepted. Suppose player 2 proposed an alternative yielding player 1 a payoff of $m_1 - \eta$, with $\eta > 0$. Then player 1 would reject it, and play would revert to the JOO-type equilibrium, yielding player 2 a payoff of m_2 immediately. Consequently, such a deviation is not profitable for player 2 if

$$m_2 \leq p^{R-1} f_2(z^*) \Rightarrow z^* \leq f_1\left(\frac{m_2}{p^{R-1}}\right)$$

which is implied by condition 5. More generally, a deviant offer at any odd round r is not going to be profitable for player 2 as long as $z^* \leq f_1(m_2/p^{R-r})$, which is implied by the upper bound on z^*. Finally, note that at any odd round r (where $r \geq 1$) player 2 can never profit by proposing an alternative yielding player 1 a payoff $z > m_1$ (which player 1 would accept), since otherwise he would be left with the residual payoff

$$f_2(z) < f_2(m_1).$$

But

$$z^* \leq f_1\left(\frac{m_2}{p^R}\right) \Leftrightarrow p^{R-r} f_2(z^*) \geq \frac{m_2}{p^r}$$

and

$$\frac{m_2}{p^r} \geq f_2(m_1)$$

which holds true since $m_2 \geq pf_2(m_1) \geq p^r f_2(m_1)$, so that player 2 cannot improve on his equilibrium payoff $p^{R-r} f_2(z^*)$.

(ii') *Deviant response to proposals to take the JOO.* Suppose, after rejecting player 2's offer, player 1 unexpectedly proposes to take the JOO, triggering his own punishment state. Then, it is optimal for player 2 to punish this deviation by rejecting the JOO, since in this way he obtains one period later his maximum payoff, $\rho_2^2 \geq 1/p\, m_2$, which holds true by definition. Similarly for odd rounds other than the second.

To conclude the proof, notice that there always exists an m such that the interval $\left[\rho_1^1/p^R, f_1(m_2/p^R)\right]$ is not empty and the conditions for the proposition hold. For instance, consider m such that $m_1 > \rho_1^1$ and $m_2 = pf_2(m_1)$, which is compatible with $m_2 < p\rho_2^2$. Then the upper bound for z^* becomes

$$f_1\left[\frac{pf_2(m_1)}{p^R}\right] = f_1\left[\frac{f_2(m_1)}{p^{R-1}}\right].$$

When $p \rightarrow 1$ the above expression tends to $m_1 > \rho_1^1$. ■