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Technical Appendix to

SOLOW AND HETEROGENEOUS LABOUR: A NEOCLASSICAL EXPLANATION OF WAGE INEQUALITY

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Appendix

This Appendix contains the proof of Lemma 3. The proof makes use of the following lemma:

Lemma A1 *The outcome of the competitive economy is equivalent to the outcome in a command economy where a social planner chooses an allocation that maximises aggregate output.*

We proof this equivalence first.

Proof. A social planner that tries to maximise aggregate output has to decide how to allocate given K between production and human capital acquisition (i.e. he has to choose how much capital K_P to apply as a direct input to production and how much capital K_I to use for education). His problem is

$$\max_{K_I, K_P}\left\{F(K_P, H) : K \geq K_P + K_I, K_I = (\overline{a} - a_1)/(\overline{a} - \underline{a})I,\right.$$

$$H = \int_{\underline{a}}^{a_1} \frac{a}{\overline{a} - \underline{a}}\mathrm{d}a + \int_{a_1}^{\overline{a}} \frac{a^2}{\overline{a} - \underline{a}}\mathrm{d}a,$$

$$\left. K_P \geq 0, 0 \leq K_I \leq I\right\}$$

where a_1 denotes the threshold ability implied by maximising behaviour. That optimisation problem can be recast as

$$\max_{a_1}\left\{F\left(K - \frac{\overline{a} - a_1}{\overline{a} - \underline{a}}I, \frac{1}{2}\frac{a_1^2 - \underline{a}^2}{\overline{a} - \underline{a}} + \frac{1}{3}\frac{\overline{a}^3 - a_1^3}{\overline{a} - \underline{a}}\right) : \underline{a} \leq a_1 \leq \overline{a}\right\}.$$

Some simple calculations show that for $K \leq \underline{K}$ no physical capital is used for additional qualification of the workers, since in this case $F_K(K, \underline{H}) \geq F_H(K, \underline{H})(\overline{a}^2 - \overline{a})/I$ according to the definition of $\underline{K}$. This result is analogous to the corresponding competitive equilibria in which there are no skilled workers. For $K \geq \overline{K}$ all human capital investment opportunities are exploited and the dictator uses additional capital as a direct production input. For $K \in (\underline{K}, \bar{K})$, the above maximisation problem has an interior solution with a_1 implicitly given by

$$F_K\left(K-\frac{\overline{a}-a_1}{\overline{a}-\underline{a}}I,\frac{1}{2}\frac{a_1^2-\underline{a}^2}{\overline{a}-\underline{a}}+\frac{1}{3}\frac{\overline{a}^3-a_1^3}{\overline{a}-\underline{a}}\right)$$
$$-F_H\left(K-\frac{\overline{a}-a_1}{\overline{a}-\underline{a}}I,\frac{1}{2}\frac{a_1^2-\underline{a}^2}{\overline{a}-\underline{a}}+\frac{1}{3}\frac{\overline{a}^3-a_1^3}{\overline{a}-\underline{a}}\right)(a_1^2-a_1)/I = 0$$

Obviously this corresponds to a competitive equilibrium with

$$k = \frac{K-\dfrac{\overline{a}-a_1}{\overline{a}-\underline{a}}I}{\dfrac{1}{2}\dfrac{a_1^2-\underline{a}^2}{\overline{a}-\underline{a}}+\dfrac{1}{3}\dfrac{\overline{a}^3-a_1^3}{\overline{a}-\underline{a}}},$$

since $1+r=f'(k)$ and $w=f(k)-kf'(k)$ imply a threshold ability a_1. Owing to the uniqueness of competitive equilibria for given K, the equivalence is proven.

We are now able to prove Lemma 3.

Proof. Owing to the equivalence of the competitive and the command environment, the continuity between K and k (see Corollary 1) holds in the command economy, as well. For $K>0$, the threshold ability above which the planner chooses to upgrade human capital is given by the $a_s(K)$ (the corresponding function a_s was introduced in (6)). An additional unit of capital yields more output according to $F_K[K-(\overline{a}-a_1)(\overline{a}-\underline{a})I,H(a_1)]>0$ (the envelope theorem).[1] The equivalence between the environments implies that the marginal product equals $f'[g(K)]$. Concavity of the per-capita production function f and the monotonicity of g prove the claim.

[1] For $a_1 \in [\underline{a},\overline{a}]$ this is due to the fact that the margin capital is equally useful in both applications.