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Technical Appendix to

THE SELF-PERPETUATION OF BIASED BELIEFS*

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Appendix

A. *Proof of Proposition* 2

First, note that (9) can be simplified to

$$\mathrm{E}V = \pi - \{\pi[1 - F_1(t)] + (1-\pi)[1 - F_2(t)]\}\pi_{up}^2 - [\pi F_1(t) + (1-\pi)F_2(t)]\pi_{down}^2.$$

Take derivative of $\mathrm{E}V$ with respect to t to get

$$\begin{aligned}\frac{\partial \mathrm{E}V}{\partial t} &= [\pi f_1 + (1-\pi)f_2]\left(\pi_{up}^2 - \pi_{down}^2\right)\\ &\quad - 2\pi_{up}\frac{-\pi f_1[\pi(1-F_1) + (1-\pi)(1-F_2)] + [\pi f_1 + (1-\pi)f_2]\pi(1-F_1)}{\pi(1-F_1) + (1-\pi)(1-F_2)}\\ &\quad - 2\pi_{down}\frac{\pi f_1[\pi F_1 + (1-\pi)F_2] - [\pi f_1 + (1-\pi)f_2]\pi F_1}{\pi F_1 + (1-\pi)F_2}\\ &= [\pi f_1 + (1-\pi)f_2]\left(\pi_{down}^2 - \pi_{up}^2\right) + 2\pi f_1(\pi_{up} - \pi_{down})\\ &= [\pi f_1 + (1-\pi)f_2](\pi_{up} - \pi_{down})(2\pi_l - \pi_{up} - \pi_{down}),\end{aligned}$$

where $\pi_l = \pi f_1/[\pi f_1 + (1-\pi)f_2]$. Therefore, the first-order condition for minimising $\mathrm{E}V$ can be written as

$$FOC = 2\pi_l - \pi_{up} - \pi_{down} = 0.$$

Note that $\pi_{up} = rL_{**}/(rL_{**} + 1)$, $\pi_{down} = rL_*/(rL_* + 1)$, and $\pi_l = rl/(rl + 1)$. Then

$$\begin{aligned}\frac{\partial FOC}{\partial r} &= \frac{2l}{(rl+1)^2} - \frac{L_{**}}{(rL_{**}+1)^2} - \frac{L_*}{(rL_*+1)^2}\\ &= \frac{1}{r}\left(\frac{2\pi_l}{rl+1} - \frac{\pi_{up}}{rL_{**}+1} - \frac{\pi_{down}}{rL_*+1}\right).\end{aligned}$$

Using the first-order condition $2\pi_l = \pi_{up} + \pi_{down}$, the expression in parenthesis can be written as

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$$\begin{aligned}&\pi_{up}\left(\frac{1}{rl+1}-\frac{1}{rL_{**}+1}\right)+\pi_{down}\left(\frac{1}{rl+1}-\frac{1}{rL_{*}+1}\right)\\&\quad=\pi_{up}(\pi_{up}-\pi_l)+\pi_{down}(\pi_{down}-\pi_l)\\&\quad=(\pi_{up}-\pi_{down})(\pi_{up}-\pi_l)\\&\quad>0.\end{aligned}$$

Hence $\partial FOC/\partial r > 0$. The second-order condition requires that $\partial FOC/\partial t > 0$. By the implicit function theorem, one obtains $\partial t^*/\partial r < 0$.

B. *Proof of P2*

(a) Suppose $Y < t_A$. Then $r' = rL_*[t^*(r, c)]$. Using the result $\mathrm{d}t^*/\mathrm{d}r = -l/rl'$, it can be seen that $\mathrm{d}r'/\mathrm{d}r = L_* - rL_*'(l/rl') > 0$ if l/L_* is an increasing function. Therefore, $r'_A > r'_B$. (b) Suppose $t_A \le Y < t_B$. Then $r'_A > r_A > r_B > r'_B$. (c) Suppose $Y \ge t_A$. Then $r' = rL_{**}[t^*(r, c)]$. As in case (a), $\mathrm{d}r'/\mathrm{d}r_i > 0$ if l/L_{**} is an increasing function. Therefore $r'_A > r'_B$.

C. *Proof of P3*

By Jensen's inequality,

$$\begin{aligned}\mathrm{E}(z \mid s_2) &= F_2(t)\log L_*(t) + [1-F_2(t)]\log L_{**}(t)\\&< \log\{F_2(t)L_*(t) + [1-F_2(t)]L_{**}(t)\}\\&= 0.\end{aligned}$$

For the second statement, since an individual with higher r chooses an advisor with lower t, it suffices to determine how t affects $\mathrm{E}(z)$. From the equation above,

$$\begin{aligned}\frac{\mathrm{dE}(z \mid s_2)}{\mathrm{d}t} &= f_2(\log L_* - \log L_{**}) + F_2\frac{1}{L_*}\frac{f_2}{F_2}(l-L_*) + (1-F_2)\frac{1}{L_{**}}\frac{f_2}{1-F_2}(L_{**}-l)\\&= f_2\left[\left(\frac{l}{L_*}-1-\log\frac{l}{L_*}\right)-\left(\frac{l}{L_{**}}-1-\log\frac{l}{L_{**}}\right)\right].\end{aligned}$$

Note that the function $g(x) = x - 1 - \log x$ is non-negative for all x. As t approaches $\underline{y}$, l/L_* approaches 1 and $\mathrm{dE}(z \mid s_2)/\mathrm{d}t = -f_2(t)g(l/L_{**}) < 0$. As t approaches $\bar{y}$, l/L_{**} approaches 1 and $\mathrm{dE}(z \mid s_2)/\mathrm{d}t = f_2(t)g(l/L_*) > 0$. Therefore there exists some intermediate value $\hat{t}$ such that $\mathrm{dE}(z \mid s_2)/\mathrm{d}t = 0$ when $t = \hat{t}$. Furthermore, since $g(x)$ is decreasing for $x < 1$ and is increasing for $x > 1$, and since l/L_* and l/L_{**} are both increasing in t, we have $\mathrm{dE}(z \mid s_2)/\mathrm{d}t < 0$ for all $t < \hat{t}$ and $\mathrm{dE}(z \mid s_2)/\mathrm{d}t > 0$ for all $t > \hat{t}$. Thus the absolute value of $\mathrm{E}(z \mid s_2)$ falls as t diverges from $\hat{t}$ in either direction.