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Technical Appendix to

IDENTIFYING THE COMMON COMPONENT OF INTERNATIONAL ECONOMIC FLUCTUATIONS: A NEW APPROACH*

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Appendix A: A simple and highly stylised illustrative example

This appendix provides a example to motivate the methodology for constructing time-varying weights. Consider the following simple model:

$$y_{it} = \alpha_i u_t + \varepsilon_{it}$$

where y_{it} is country *i's IP* growth rate, which can be decomposed into a piece based on the common component u_t and an idiosyncratic component ε_{it}. In vector form, the model is written as:

$$\mathbf{Y}_t = \boldsymbol{\alpha}\mathbf{U}_t + \boldsymbol{\varepsilon}_t$$

where $\mathbf{Y}_t$, $\boldsymbol{\alpha}$ and $\boldsymbol{\varepsilon}_t$ are of dimension $n \times 1$, and $\mathbf{U}_t$ is of dimension 1×1. Also, as in a standard factor model, assume that $\mathbf{U}_t$ and $\boldsymbol{\varepsilon}_t$ are independent. The variance covariance matrix is given by

$$\mathrm{E}\left(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'\right) = \mathbf{D} = \left[d_{ii,t}\right] \text{ where } d_{ii,t} = f(y_{it-1})$$

that is, its elements are functions of lagged IP growth, which vary across equations. Now consider the special case where $\boldsymbol{\alpha}$ = an $n \times 1$ vector of ones and the variance of u_t, denoted σ_u^2, is constant. The least squares estimator of u_t is then given by

$$\hat{u}_t = \frac{1}{n}\sum_{i=1}^{n} y_{it}.$$

This estimator is not optimal, however, due to the heteroscedasticity of ε_t. In this case, GLS is optimal and is equivalent to the LS estimation of $\tilde{y}_{it} = \tilde{\alpha}_i u_t + \tilde{\varepsilon}_{it}$ where $\tilde{\varepsilon}_{it}$ is spherical, $\tilde{y}_{it} = y_{it}/\sqrt{d_{ii,t}}$ and $\tilde{\alpha}_i = 1/\sqrt{d_{ii,t}}$. Thus, $\hat{u}_t = \sum_{i=1}^{n} w_i y_{it}$, where the weights are given by

$$\frac{\tilde{\alpha}_i}{\mathrm{TR}(\mathbf{D})} = \frac{1}{\sqrt{d_{ii,t}}} \Bigg/ \sum_{i=1}^{n} \frac{1}{d_{ii,t}}$$

and TR($\mathbf{D}$) is the trace of the matrix $\mathbf{D}$.

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Appendix B: Numerical evidence

A possible concern with our methodology is that, if a country were to be relatively more sensitive to global shocks, its output could have higher volatility and our methodology would assign a lower weight to that country (in the construction of the common component) despite the fact that it could be more important for identifying the common component. In this Appendix, we use some simulation experiments to address the possible empirical importance of this issue.

Consider a two-country version of the simple model set out in Appendix A:

$$y_{it} = \alpha_i u_t + \varepsilon_{it} \quad \forall i = 1, 2$$

where y_{it} is country *i's* IP growth rate, which can be decomposed into a piece based on the common component u_t and an idiosyncratic component ε_{it}. The shocks u_t, ε_{1t} and ε_{2t} are assumed to be serially and mutually uncorrelated. Further, the two countries are assumed to be of equal economic size.

In the baseline case, $\alpha_1 = \alpha_2$ and $\text{var}(\varepsilon_{1t}) = \text{var}(\varepsilon_{2t})$. Since $\text{var}(y_{1t}) = \text{var}(y_{2t})$, it follows that the average weights for the two countries are both equal to 50.0 (weights are normalised to sum to 100). Now consider the case where $\alpha_2 > \alpha_1$, implying that $\text{var}(y_{2t}) > \text{var}(y_{1t})$. Our methodology would mechanically assign a lower average weight to country 2 since it has more volatile output fluctuations, even though the additional volatility is attributable to greater sensitivity to the common shock. The question that we address below is: empirically, how important is this potential problem? To simplify the analysis, we focus below only on *average* weights for each country.

Let $\alpha_1 = 1$, $\alpha_2 = 3$ and $\text{var}(u_t) = \text{var}(\varepsilon_{1t}) = \text{var}(\varepsilon_{2t}) = 0.002$. This is a rather extreme case (as discussed below) since it assumes that the contribution of the common component to each country's output fluctuations is as large as (country 1) or three times larger than (country 2) that of country-specific shocks. For country 1, this implies a level of unconditional volatility (standard deviation of about 0.06) near the median volatility for the countries in our sample (Table B1 shows standard deviations of IP growth for each country). For country 2, the resulting volatility is just above the upper end of the range of observed volatilities. With this parameterisation, the resulting average weights for the two countries would be 69.1 and 30.9, respectively, suggesting a possibly significant 'distortion' relative to equal weighting.

However, the common component (CC) typically has much lower relative volatility than in this example. For instance, the standard deviation of the CC that we estimate is about half the standard deviation of US output growth (the country with the lowest volatility), which also implies that it is substantially less volatile than output fluctuations in other countries. Furthermore, in the variance decompositions from the VARs discussed in Section 2.3 of the printed paper, we note that, for most countries, the CC generally contributes at most 25–30% of the forecast error variance of domestic output fluctuations.[1]

Based on these results, we now reparameterise the two-country example more realistically. First, we change the variance of the CC such that $\text{var}(u_t) = 0.3\ \text{var}(\varepsilon_{1t}) = 0.3\ \text{var}(\varepsilon_{2t})$ and scale up the variances of the idiosyncratic shocks so that the variance of y_{1t} is again 0.06. Next, to make a more realistic assumption about the relative contribution of the CC to output fluctuations, we set $\alpha_1 = 0.25$ and $\alpha_2 = 3\alpha_1 = 0.75$. The average weights for the two countries in the construction of the CC are then 51.6 and 48.4, respectively, suggesting only a small distortion relative to the equal-weight benchmark.

[1] Other studies using different techniques have also found that, notwithstanding the importance of the common component, global factors still account for a much smaller fraction of output fluctuations than country-specific factors and also have lower volatility; see, e.g., Stockman (1988), Gregory *et al.* (1997).

Table B1

Descriptive Statistics for and Time Series Properties of Industrial Production Indexes

	Annualised Mean Growth Rates (%)			Standard Deviation			
	Full Sample	1963–73	1974–94	Full Sample	1963–73	1974–94	Box-Pierce Q-statistic
Austria	3.64	5.92	2.54	0.036	0.031	0.029	103.66
Belgium	2.50	4.94	1.18	0.040	0.029	0.035	80.51
Canada	3.61	6.91	1.83	0.021	0.019	0.017	85.70
Finland	4.37	6.76	3.34	0.045	0.022	0.043	349.29
France	2.72	5.69	1.09	0.041	0.037	0.029	157.99
Greece	4.91	10.09	1.65	0.044	0.030	0.039	42.21
Germany	2.37	4.93	0.90	0.038	0.032	0.036	39.38
Italy	3.14	5.99	1.67	0.091	0.039	0.059	404.00
Japan	5.44	11.47	2.47	0.022	0.016	0.021	94.73
Luxembourg	2.01	3.23	1.41	0.065	0.024	0.059	223.78
Norway	5.44	5.30	5.13	0.082	0.056	0.083	99.91
Netherlands	3.51	7.34	1.04	0.041	0.025	0.041	31.20
Portugal	4.52	5.90	3.65	0.079	0.051	0.051	286.67
Spain	4.61	10.43	1.76	0.085	0.039	0.059	337.59
Sweden	2.47	5.19	1.25	0.073	0.034	0.084	311.35
UK	1.84	3.17	1.08	0.034	0.026	0.036	60.51
US	3.35	5.46	2.22	0.012	0.008	0.011	42.75

Notes: The descriptive statistics reported in the first six columns of this table are for data that were transformed into logarithms, first differenced, and then deseasonalised by regressing on a set of monthly dummies. The annualised mean growth is calculated as 100 [(1 + *MEAN*) × 12] − 100, where *MEAN* is the sum of the coefficients on the deterministic seasonals in the deseasonalising regression. Using residuals from a regression of IP growth on a constant and 12 lags of IP growth, the Box–Pierce Q-statistics for the squared residuals were computed using twelve sample autocorrelations. Under the null, this statistic is distributed as chi-squared with 12 degrees of freedom. The 1% critical value for this test statistic is 26.2.

We also performed some simulations with a multi-country model with $\mathrm{var}(u_t) = 0.3\,\mathrm{var}(\varepsilon_{it})$ and $\mathrm{var}(\varepsilon_{it})$ the same for all *i*. We included 17 countries, as in our full sample, and experimented with different degrees of dispersion of α_i (these were generated from a uniform distribution).When $\mathrm{var}(\alpha_i) = 0$, the average weights are equal $((1/17)100 = 5.88)$. As $\mathrm{var}(\alpha i)$ increases, the dispersion of these weights increases. When we set $\mathrm{mean}(\alpha_i) = 0.25$ and $\mathrm{var}(\alpha_i) = 0.02$, the parameter α ranges from 0.03 to 0.47 and the weights range from 5.76 to 5.95. Only in a somewhat extreme case, where $\mathrm{mean}(\alpha_i) = 0.4$, $\mathrm{var}(\alpha_i) = 0.08$ and α goes from 0.06 to 0.94, do the weights differ significantly from the equal-weight benchmark, ranging in this case from 5.46 to 6.15.

These examples (and numerous others that we looked at) lead us to the general proposition that, as the cross-sectional variance of α_i rises, the potential distortion from our weighting methodology increases. However, this effect is substantially mitigated by (i) the larger variance of country-specific shocks relative to that of the CC and (ii) the greater contribution of country-specific shocks, rather than the CC, to output fluctuations in each country. Thus, for this paper's application of our methodology for constructing time-varying weights, the distortion is likely to be quite small.

Appendix C: Data used in the analysis

This Appendix briefly describes the data used and elaborates on some of the issues discussed in Section 1 of the printed paper, including that of seasonality.

Monthly indices of industrial production (not seasonally adjusted) for 17 OECD economies over the period 1963-94 were taken from the OECD Analytical Database.[2] The data are transformed into logarithms and first differenced to achieve stationarity and are then seasonally adjusted by regressing the log differences on 12 monthly dummy variables. We choose to take first differences in part because, as noted by Baxter and Stockman (1989), this procedure 'emphasises the higher frequencies associated with business cycles' relative to linear detrending. Table B1 provides summary statistics for the data over the full sample and also for the pre- and post-1973 subsamples.

We tested the hypothesis that the raw data are difference stationary by testing for the presence of a unit root in the logarithms of the data using Augmented Dickey-Fuller regressions with twelve monthly seasonal dummy variables included. We found that in only one case was the unit root hypothesis rejected in favour of trend-stationarity – the US. This is somewhat at odds with previous results for the US; for example, Nelson and Plosser (1982) did not reject the unit root hypothesis for industrial production using annual data from 1869–1970. Gerlach (1988), who used industrial production data for 1963:9–1986:3, also finds little evidence against the unit root hypothesis for the countries in his sample, including the US. Hence, we take first differences so as to transform the data for all countries in a uniform manner. As a check that we have adequately purged the data of nonstationarity, we also tested the differenced data for the presence of a unit root. For every country, the null hypothesis of a unit root in the first differences was rejected in favour of stationarity. The results of the stationarity tests are reported in the working paper version of this paper and are available upon request.

An important issue that arises in using seasonally unadjusted macroeconomic data is the relative importance of seasonal fluctuations. Visual inspection of our monthly industrial production data indicated that there were strong seasonal components in virtually every country in our sample; these were particularly large and noticeable in countries like Italy. Further evidence is provided by time series regressions which show that deterministic seasonal dummies can explain a substantial fraction of the variation in monthly industrial production growth rates for most countries.[3]

The appropriate treatment of seasonal effects is, however, fraught with complications. A simple procedure adopted by many authors; e.g., Beaulieu and Miron (1992) Beaulieu *et al.* (1992), is to regress the unadjusted data on seasonal dummies. Other deterministic filters such as the Census Bureau's X-11 procedure have also been used widely, although it has been argued that such filters do not necessarily retain the salient features of the data; e.g., Ghysels and Perron (1993). On the other side of the debate are authors such as Franses *et al.* (1995) who argue that stochastic seasonality in the form of seasonal unit roots is the appropriate characterisation of seasonal fluctuations. These authors recommend seasonal differencing in order to eliminate unit roots at seasonal frequencies.

As mentioned in the text, we prefer to remain agnostic on the appropriate characterisation of seasonal variation in the data. Hence, we deal with seasonality only to the extent that it could potentially interfere with identification. As a practical matter, we take out only the deterministic seasonal component by regressing the raw data on 12 monthly dummies and using the residuals in our empirical work. In Section 3.3 of the printed paper, we test the robustness of our results to this transformation by using seasonally unadjusted data.

[2] See, e.g., Beaulieu and Miron (1992), Beaulieu *et al.* (1992), Canova and Ghysels (1994), Cecchetti and Kashyap (1996), and Cecchetti *et al.* (1997).

[3] For the countries in our sample, regressions on seasonal dummy variables indicated that, on average, about 80% of the variation in log differences of unadjusted monthly industrial production could be explained by these seasonal factors. The R^2 from these regressions ranged from 53% for Greece to 95% for Sweden. In most cases, the seasonal effects remained as important even when quarterly averages of the unadjusted data were used.

References

Altonji, J. and Ham, J. C. (1990). 'Variation in employment growth in Canada: the role of external, national, regional and industrial factors', *Journal of Labor Economics*, vol. 8, pp. S198–236.

Artis, M. J. and Zhang, W. (1999). 'Further evidence on the international business cycle and the ERM: is there a European business cycle?', *Oxford Economic Papers*, vol. 51, pp. 120–32.

Backus, D. K. and Kehoe, P. J. (1992). 'International evidence on the historical properties of business cycles', *American Economic Review*, vol. 82, pp. 864–88.

Baxter, M. and Stockman, A. C. (1989). 'Business cycles and the exchange rate regime: some international evidence', *Journal of Monetary Economics*, vol. 23, pp. 377–400.

Bayoumi, T. and Prasad, E. S. (1997). 'Currency unions, economic fluctuations, and adjustment: some new empirical evidence', *IMF Staff Papers*, vol. 44, pp. 36–58.

Beaulieu, J. J. and Miron, J. A. (1992). 'A cross-country comparison of seasonal cycles and business cycles', ECONOMIC JOURNAL, vol. 102, pp. 772–88.

Beaulieu, J. J., MacKie-Mason, J. K. and Miron, J. A. (1992). 'Why do countries and industries with large seasonal cycles also have large business cycles?', *Quarterly Journal of Economics*, vol. 107, pp. 621–56.

Bollerslev, T. (1986). 'Generalized autoregressive conditional heteroskedasticity', *Journal of Econometrics*, vol. 31, pp. 307–27.

Bollerslev, T. (1990). 'Modeling the coherence in short-run nominal exchange rates: a multivariate generalized ARCH model', *Review of Economics and Statistics*, vol. 72, pp. 498–505.

Canova, F. and Ghysels, E. (1994). 'Changes in seasonal patterns: are they cyclical?', *Journal of Economic Dynamics and Control*, vol. 18, pp. 1143–71.

Cecchetti, S. G. and Kashyap, A. K. (1996). 'International cycles', *European Economic Review*, vol. 40, pp. 331–60.

Cecchetti, S. G., Kashyap, A. K. and Wilcox, D. W. (1997). 'Interactions between the seasonal and business cycles in production and inventories', *American Economic Review*, vol. 87, pp. 884–92.

Christofferson, P. F. and Giorgianni, L. (2000). 'Interest-rate arbitrage in currency baskets: forecasting weights and measurement risk', *Journal of Business and Economic Statistics*, vol. 18, pp. 242–53.

Deutsch, M., Granger, C. W. J. and Teräsvirta, T. (1994). 'The combination of forecasts using changing weights', *International Journal of Forecasting*, vol. 10, pp. 47–57.

Diebold, F. X. and Nerlove, M. (1989). 'The dynamics of exchange rate volatility: a multivariate latent factor ARCH model', *Journal of Applied Econometrics*, vol. 4, pp. 1–21.

Diebold, F. X. and Pauly, P. (1990). 'Use of prior information in forecast combination', *International Journal of Forecasting*, vol. 6, pp. 503–8.

Diebold, F. X. and Rudebusch, G. D. (1996). 'Measuring business cycles: a modern perspective', *Review of Economics and Statistics*, vol. 78, pp. 67–77.

Engle, R. F. (1982). 'Autoregressive conditional heteroskedasticity', *Econometrica*, vol. 50, pp. 987–1008.

Engle, R. F. and Hylleberg, S. (1996). 'Common seasonal features: global unemployment', *Oxford Bulletin of Economics and Statistics*, vol. 58, pp. 615–30.

Engle, R. F. and Kozicki, S. (1993). 'Testing for common features', *Journal of Business and Economic Statistics*, vol. 11, pp. 369–95.

Forni, M. and Reichlin, L. (1996). 'Dynamic common factors in large cross-sections', *Empirical Economics*, vol. 21, pp. 27–42.

Franses, P. H., Hylleberg, S. and Lee, H. S. (1995). 'Spurious deterministic seasonality', *Economics Letters*, vol. 48, pp. 249–56.

Gerlach, S. (1988). 'World business cycles under fixed and flexible exchange rates', *Journal of Money, Credit, and Banking*, vol. 20, pp. 621–32.

Ghysels, E. (1993). 'Seasonal adjustment and other data transformations', Univérsite de Montréal Working Paper.

Gregory, A. W., Head, A. C. and Raynauld, J. (1997). 'Measuring world business cycles', *International Economic Review*, vol. 38, pp. 677–702.

Hamilton, J. D. (1989). 'A new approach to the economic analysis of nonstationary time series and the business cycle', *Econometrica*, vol. 57, pp. 357–84.

Head, A. C. (1995). 'Country size, aggregate fluctuations and international risk sharing', *Canadian Journal of Economics*, vol. 28, pp. 1096–119.

Hong, C. (1987). 'The integrated generalized autoregressive conditional heteroskedasticity model: the process, estimation and some Monte Carlo experiments', unpublished manuscript, University of California, San Diego.

Lippi, M. and Reichlin, L. (1994). 'Common and uncommon trends and cycles', *European Economic Review*, vol. 38, pp. 624–35.

Lumsdaine, R. L. (1995). 'Finite sample properties of the maximum likelihood estimator in GARCH(1,1) and IGARCH(1,1) models: a Monte Carlo investigation', *Journal of Business and Economic Statistics*, vol. 13, pp. 1–10.

Lumsdaine, R. L. (1996). 'Consistency and asymptotic normality of the IGARCH(1,1) and covariance stationary GARCH(1,1) models', *Econometrica*, vol. 64, pp. 575–96.
Lumsdaine, R. L. and Ng, S. (1999). 'Testing for ARCH in the presence of a possibly misspecified conditional mean', *Journal of Econometrics*, vol. 83, pp. 257–59.
Nelson, C. R. and Plosser, C. I. (1982). 'Trends and random walks in macroeconomic time series: some evidence and implications', *Journal of Monetary Economics*, vol. 10, pp. 139–62.
Norrbin, S. and Schlagenhauf, D. (1996). 'The role of international factors in the business cycle: a multicountry study', *Journal of International Economics*, vol. 40, pp. 85–104.
Pagan, A. (1997). 'Towards an understanding of some business cycle characteristics', *The Australian Economic Review*, vol. 30, pp. 1–15.
Stock, J. H. and Watson, M. W. (1989). 'New indexes of coincident and leading economic indicators', in (O. Blanchard and S. Fischer, eds), *NBER Macroeconomics Annual*, Cambridge, MA: MIT Press, pp. 351–94.
Stock, J. H. and Watson, M. W. (1993). 'A procedure for predicting recessions with leading indicators: econometric issues and recent experience', in (J.H. Stock and M.W. Watson, eds), *Business Cycles, Indicators and Forecasting*, Chicago: University of Chicago Press, pp. 255–84.
Stockman, A. (1988). 'Sectoral and national aggregate disturbances to industrial output in seven European countries', *Journal of Monetary Economics*, vol. 21, pp. 387–409.